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NOTES

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BY

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ADVERTISEMENT.

In submitting to the notice of the naval profession a new edition of "Notes on the Management of Chronometers, and the Measurement of Meridian Distances," the Author has been desirous of rendering this work still more worthy of the favourable attention of students of nautical science.

The whole work has been carefully revised, and many small improvements introduced. The writings of recent French authors, Givry, De Cornulier, Lieussou, Mouchez, Vincendon Dumoulin, Coupvent Desbois, and Charles Ploix, have been carefully examined, and many valuable extracts from their works, taken chiefly from the pages of the "Recherches Chronométriques," a most useful publication, now in course of issue, under the auspices of the Minister of Marine at Paris, enrich the present volume both in the text and notes.

The important questions of the effects of change of temperature, and the influence of the acceleration, have been fully entered into. For this purpose a new chapter (chapter vi.) has been interpolated in this edition, in which the systems of De Cornulier, Lieussou, and Mouchez have been amply discussed and commented on. The system proposed by Hartnup, of adopting a series of "tabulated rates," obtained by direct experiment, corresponding to the different degrees of the thermo-

metric scale, instead of a fixed daily rate, regardless of temperature, as usually employed, has also been introduced to the notice of the reader, and carefully examined.

The additional matter introduced by these changes, amounting to seventy pages of letterpress, has had the effect of altering the titles of the final chapters, chapters vii. viii. ix. and x. of this edition, respectively corresponding to chapters vi. vii. viii. and ix. of the original one. All the alterations in the work consist of additions; all the matter given formerly being still retained. Notwithstanding these changes, and consequent increase of expense in printing, the low price at which the book was originally offered to the public remains unchanged.

The Author trusts that the additions to his work, above alluded to, may prove useful to his professional brethren, and that in placing before British seamen the recent researches and analytical investigations of contemporary French writers, he may have rendered some essential service to the progressive improvement of nautical science.

Researches into the laws which regulate the changes caused by variations of temperature, and the influence of the acceleration, as a function of the time, seem to be the point towards which the improvement of chronometric science is at present tending; we trust that English navigators will take their share in contributing to the development of our knowledge on these important questions. With these hopes the Author submits his labours to the notice of the naval profession, and to the attention of students of chronometric science, trusting that they may be found worthy of their favourable consideration, and may be deemed a useful contribution to nautical knowledge.

Slough, March 1861.

PREFACE TO THE FIRST EDITION.

The important services rendered by chronometers in the ordinary course of navigation at sea, by facilitating the daily determination of the ship's position in longitude, are well understood and fully appreciated by all intelligent seamen; but in addition to the useful ends to which, under ordinary circumstances, they are subservient in the daily conduct of the ship's affairs, they are susceptible, when placed in intelligent hands, of being applied to higher scientific uses, and when rightly employed, are capable of affording valuable contributions towards the gradual perfection of Maritime Geography.

The ordinary treatises on navigation, in use among seamen, contain ample rules and directions for the application of chronometers to the common purposes of determining the longitude at sea; but the information they afford relative to the accurate and systematic measurement of "chronometric differences of longitude," or "meridian distances," is for the most part of a very meagre and insufficient character.

The points involved in the discussion of meridian distances are not in themselves difficult or abstruse, and present but few impediments to those who are expert in computation and skilled in the ordinary processes of navigation; but at the

same time, in order to obtain from the use of chronometers all the precision of results of which they are susceptible, and in order to treat the measurement of meridian distances in a uniform, organised, and systematic manner, many minutiæ must be attended to, and many points considered, on which the usual text-books are wholly silent, and on which the ordinary experience of the navigator throws but little light. It may thus happen that persons without much previous experience, who might be possessed of some good chronometers, and were desirous of employing them to the best advantage, would probably encounter many doubts and difficulties in the execution of their design, from the fragmentary, traditional, and irregular character of much of the knowledge at present existing on this subject.

Many scattered hints, and some valuable information relative to the application of chronometers to the accurate deduction of differences of longitude, exist in many detached works, but the subject seems to require condensation on some points, amplification on others, and systematic arrangement as a whole. The object of the Author in the following pages is to attempt to remedy this existing want, and to endeavour to supply naval officers, and others entrusted with the care of chronometers, with a manual of instruction how best to use them, and how to furnish systematic results in recording the meridian distances of the several places visited during their voyages.

With this end in view, the Author has freely considered, and availed himself of, several detached ideas on this subject, interspersed among various works by previous writers; among which may be enumerated,—

Forster's Voyage; Appendix by Tiarks.
Owen on Longitude.
Voyages of Adventure and Beagle; Appendix by Fitzroy.
Belcher on Nautical Surveying.

Raper on Longitudes, "Nautical Magazine," 1839, &c.
Raper's "Practice of Navigation."
Nautical Magazine, various papers; Fisher, Bayfield, Bedford, &c.
Memoirs Ast. Soc., Vols. III., XII., &c.
Connaissance des Tems, Vols. 1835–6.
Daussy sur la Marche des Chronomètres. Paris, 1840.
Lieussou. Recherches sur les Variations de la Marche des Chronomètres, &c. Paris, 1854.
&c. &c.

Blending the ideas derived from these various sources with the results of his own experience, the Author trusts that he has succeeded in producing a little work which, dealing with the questions relating to chronometers in a methodical and regular manner, may be found of some assistance to those who may hereafter be inclined to undertake the measurement of meridian distances.

In the execution of this design the Author has collected and arranged many detached precepts relative to the custody and management of chronometers, scattered at present among many books, or only existing in a traditional form. The questions of errors and rates, usually dismissed in books on navigation with a few brief remarks, next engage attention, and are discussed with an amplification of detail more commensurate, it is hoped, with their presumed importance. A method of combining observations for rate by the "*method of least squares*" is then explained, and it is hoped that, where extreme precision is sought for, the plan developed may be found useful in practice. A systematic arrangement of the formulæ for meridian distances is then undertaken, in which the method of correcting for the variation of the rate, proposed by Tiarks, and followed by Fitzroy, Bayfield, and other eminent navigators, has been amply developed and pursued to its legitimate consequences.

Numerous examples illustrate these and the other formulæ, and the work is concluded by an Appendix containing some miscellaneous matter of a useful character.

In the arrangement of the formulæ for the combination of observations for rate, and for the various cases which can occur in practice in the measurement of meridian distances, the Author has not deemed it necessary to translate the algebraic expressions involved into their equivalent verbal precepts, in accordance with the usual custom followed by the writers on navigation; to have done so would have caused a considerable addition to the bulk of the book, without, perhaps, making the subject much clearer, while the comparative infrequency of calculations of this nature, contrasted with the daily use of the formulæ of navigation, scarcely seemed to require such a course. The persons into whose hands these pages are likely to fall, and to whom they may practically prove useful, will doubtless, in general, be perfectly conversant with the ordinary processes of navigation, and expert in numerical and logarithmic computation; while in the present state of educational acquirements among naval officers, it is reasonable to assume that they will usually possess sufficient rudimentary knowledge of algebra to be able to understand, and apply with facility, the simple algebraic expressions which occur in the course of this work; and as, moreover, there is no doubt that, after mastering the first difficulties, working from formulæ is much easier and less liable to error than the blind and tedious following of verbal precepts, the Author trusts that his judgment on this point will command the assent of his readers.

At the same time, mindful of the requirements of practical persons, and conscious of the necessity of vivifying the dulness of algebraic details by copious practical illustrations, the Author has been careful to explain the application of his formulæ and precepts by numerous examples, for the most part taken from actual experience, and he trusts he is justified in expecting

that this part of his design may be found amply to satisfy the wants of practice.

Some persons may, perhaps, be of opinion, that hydrography has already attained in most parts of the world a sufficiently exact degree of precision, and that, after the correction of a few remaining gross discrepancies, further accuracy may be considered rather as the object of scientific curiosity than of practical advantage. The number of places, however, which may be considered definitely settled to a second of time, except some of the fixed observatories, are, according to Raper, very few; and, without specifying places by name, we may remark, that much remains to be done in most parts of the world towards filling up the details connected with the positions of subordinate stations; and that, for many years to come, the leisure and opportunities of officers serving on foreign stations cannot be more usefully employed than in the accumulation of data relating to the accurate connexion in longitude of the stations they may visit during their cruises.

Before concluding these remarks, the Author would wish to express his obligations to the Rev. George Fisher, Chaplain to the Greenwich Hospital Schools, for having communicated to him the valuable formula for determining the meridian distance between two stations, by means of observations, giving the *travelling* rates of the chronometers employed; and also to Fred. J. Evans, Esq., R.N. (his former colleague in H.M.S. "Fly," during the Australian survey), for kindly placing his manuscripts at his disposal, and for many valuable suggestions during the progress of this work.

In the preparation of a work of this nature, partly treading on old ground, and partly entering on new, the Author cannot but be conscious of many imperfections, which must claim the indulgence of his readers; but he trusts that the utility of his design may in some measure compensate for its defects of execution, and that his endeavours to systematise this im-

portant subject may prove useful, not only to the scientific members of his own profession, but also to intelligent officers in the Mercantile Marine; and in this hope, he confidently commits his labours to the critical judgment of the public and to the favourable consideration of his professional readers.

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ON THE

MANAGEMENT OF CHRONOMETERS

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CHAPTER I.

INTRODUCTORY OBSERVATIONS.

THE application of chronometers to the accurate determination of "Meridian Distances," or the differences of longitude of distant stations, has usually formed an important object in the scientific voyages of modern times, from those undertaken in the last century by the illustrious Cook, to those performed in our own day by more recent navigators.

The happy invention of the Electric Telegraph, the successful accomplishment of its submarine connexion, and its application to astronomical purposes, would seem to have completely and successfully solved the problem of differences of longitude of stations which are either situated on the same continent, or, if occupying insular positions, only separated by narrow seas; and there can be little doubt that, before long, the various observatories in the British Islands and on the Continent will, by its means, be accurately linked together, and their relative positions consequently determined to the last degree of mathematical correctness. Considered as base stations, they may then be viewed as forming salient points in a network of triangles described on the surface of our globe, to which minor places

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can afterwards be conveniently referred by geodetic means, so as ultimately to combine them in one comprehensive whole, hitherto unexampled for accuracy in the annals of geographical science.

From the very nature of its invention, however, and from the probable limits to its use, interposed by the difficulties of its submarine connexion, except in narrow seas, the application of the Electric Telegraph to the question of Terrestrial Longitude will doubtless be comparatively very limited; and it must still be to the successful appliance of the ordinary means at the disposal of the navigator that we must continue to look for the final solution of questions relating to the relative longitudes of outlying stations on the ocean, and their connexion with the fixed points on the great continents.

Fortunately, at a period when the successful application of the system of galvanic signals to astronomical purposes has given a great impetus to the final solution of the questions concerning the relative longitude of stations on land, the gradual improvements effected of late years in the construction of marine chronometers, and the yearly increasing extension of the application of steam to ocean navigation, seem at the same time to afford increased facilities to the Navigator for the improvement of Maritime Geography, as he will thus be enabled also to maintain an honourable rivalry with the Astronomer, and, like him, to contribute his fair share towards the ultimate perfection of geographical science.

In examining the earlier history of modern hydrography, and on inquiring into the circumstances which have hitherto impeded its progress towards final perfection, notwithstanding the zealous and useful labours of numerous scientific navigators, and the voluntary contributions of many intelligent commanders, two causes will, we apprehend, be found to have tended, although accidentally, to retard its satisfactory development; first, the practice of mixing up in one indiscriminate combination the astronomical data for the positive settlement of disputed positions, and the relative evidence afforded by chronometric measurements; and, secondly, a want of clearness of comprehension of the relative values of absolute and differential longitudes.

For instance, nothing is more common than to find, on examining even comparatively recent works on hydrography,

that the data quoted for the settlement of a given position are of the most miscellaneous and incommensurable character: lunar observations, eclipses of Jupiter's satellites, occultations of stars by the moon, solar eclipses, and chronometric measurements by various authorities, from adjacent and independent points, all blended together in one crude and inharmonious result ; or again, than to find chronometric determinations of a purely relative character, and often measured from two or more independent stations, confounded with absolute results.*

Places are fixed absolutely by astronomical observations, or relatively by chronometers. This distinction must be clearly kept in view if ever we wish to arrive at final and conclusive results, and if we desire to avoid the perpetual oscillation of ideas which a mixture of the two principles is sure to entai on us.

Much has been done of late years towards simplifying the conclusions of maritime geography, by collecting from the records of astronomical and chronometric observations satisfactory details for the final establishment of several important fundamental positions.

M. Daussy in France, and Mr. Raper among our own countrymen, have specially distinguished themselves by the prominent part they have taken in this useful hydrographic labour.

The valuable contributions of the former writer on this important subject will be found interspersed among the pages of the "Connaissance des Tems," especially in the volumes for 1835 and 1836; and the development of the views of the latter author in various detached papers given in the "Nautical Magazine" for 1839 and subsequent years, and also in the remarks accompanying his "Table of Maritime Positions," published in the "Practice of Navigation."

Raper's views on this important subject are so clear, and his remarks so apposite, that, with the view of enforcing the system that he recommends, we shall take the liberty of quoting largely from them, feeling conscious that no independent language of our own can place the subject in a more forcible light, or more strongly illustrate the necessity of its adoption. In

* Horsburgh's "East India Directory," valuable work as it undoubtedly is, contains numerous illustrations of this mode of proceeding.

fact, we would wish it to be understood, that one principal object which we have in view in the preparation of these pages, is to facilitate the practical adoption of the plan that he recommends, and to systematise the determination of chronometric differences of longitude, by arranging, for the benefit of naval officers and others, the details of the operations in a uniform, organised, and consistent manner.

Raper's remarks on this subject are as follows:—"Previously to Cook's voyages, which may be considered as the commencement of modern hydrography, the only method (besides the rude and imperfect determination of the ship's run) of obtaining the longitude of every new land made, was the lunar observation. But as that method, from its inaccuracy,* fails altogether in exhibiting truly relative positions, chronometers were employed in combining together the results of observations taken at different places, of which numerous instances are recorded by Horsburgh in his 'East India Directory.' Since, however, the observations made at two places are not in general equally good, this method of combining observations with chronometric differences has the disadvantage of impairing the better determination of the two, and in consequence throws a difficulty over the connexion of either of them with a third place better known. Succeeding navigators proceeding in the same way have obtained other results of observation, and other chronometric differences; and, in consequence, the hydro-

* The first edition of this work here contained the following note:—

"Notwithstanding the favour with which lunar observations have been regarded by some navigators, the important service they doubtless rendered in approximately fixing positions during the earlier periods of modern hydrography, and their still inestimable value at sea, as affording to the seaman the only available independent method of determining his absolute longitude; yet we fear, that in the present condition of hydrographic science they are quite unsuited to the settlement of disputed positions. In all observations for longitude connected with the moon, the errors of observation are multiplied in their effects on the resulting longitude by a factor whose mean value is about 30; consequently an error of 10″ in a lunar distance (and we presume that, under the most favourable circumstances, we have no right to expect less) becomes 300″ or 5′ in the resulting longitude deduced from it: and this, be it observed, is wholly independent of the additional errors it may be liable to, depending on the moon's place in the tables, owing to the still imperfect state of the lunar theory." (See also Raper's remarks on this subject, "Naut. Mag." 1839, p. 319.)

The opinions expressed above may now, perhaps, require some modification, in so far as regards the accuracy of the lunar tables. From and after the year 1862, the lunar calculations in the "Nautical Almanac" will be based on Hansen's new and

grapher who has not the means afforded him of instituting a critical examination of the several positions, or of their connexion with each other, is driven to the necessity of taking a mean between each new result and those adopted from former navigators, and thus the whole mass of positions is kept in a state of perpetual fluctuation, from which it is impossible that universal precision can ever be obtained.

"In marine surveys again, different meridians have been assumed, and different longitudes of the same meridian. In some cases the longitude of the meridian assumed has not been given; in others, the meridian itself has not been specified at all.

"If, however, instead of thus throwing open the discussion of every place at each new voyage of discovery, or surveying expedition, and unsettling all that had previously been done, without any assurance that the new series of positions would not in its turn be unsettled again, navigators and hydrographers would agree to *consider*, for the time being only, certain important stations as already established in longitude, whether really so or not, with the view of referring all the subordinate positions to them, the indistinctness which now hangs over absolute and relative positions would be forthwith cleared up. The question would be narrowed into the determination of *chronometric differences* alone, until favourable opportunities occurred for the definitive determination of a fundamental position. Accurate chronometric measures would be no longer lost to the

improved tables, in the construction of which the highest resources of analytical talent, and the profoundest mathematical ability, have been successfully employed. A very high authority has recently declared, that "the residual errors of the lunar theory are now reduced to almost insignificant limits, certainly within the errors incidental to meridional observation with first-rate instruments, and very far within the limits of accuracy of observation with the sextant;" that "Hansen's tables are practically perfect for all the purposes of navigation, and that the great nautical problem of finding the longitude at sea is now completely solved."

Under these improved and altered circumstances, it is possible that a careful series of lunar observations, for the determination of the longitude of any specific station, might repay the labour bestowed on them. By making the series sufficiently numerous, embracing an equal number of observations on each side of the moon—east and west—and taking them with every regard to accuracy, the instrumental errors of observation would be either eliminated or very materially reduced. Such a series of observations, however, would require very careful reduction, attention to the corrections for the earth's spheroidity, and taking into account the barometric and thermometric corrections to the mean refraction, due to the actual state of the atmosphere at the time of observation.

world by being merged in the uncertain results of a few astronomical observations; and the labours of each navigator would always maintain their proper value, instead of being set aside, as they must inevitably be, on the appearance of a new survey, in which the data are exhibited in a distinct form. The works of different navigators, and of the navigators of different countries, could be brought into immediate comparison,—a task which is at present often difficult and unsatisfactory, if not impossible. The labours of the hydrographer would be materially simplified; and as the points to which inquiry should next be directed would by this system be distinctly brought into view, the whole subject would advance steadily to its ultimate perfection."*

In furtherance of these views, it was accordingly proposed ("Naut. Mag." 1839, p. 399) "to adopt certain points under the name of *Secondary Meridians*, this general term being used to distinguish them from the *prime* Meridians, as Greenwich, Paris, &c., from which the longitudes in the tables or on the charts must be reckoned. The number of these stations at present proposed to be employed is twenty, but this is altogether matter of convenience, and will vary with the progress of the subject. The points selected are so far distant from each other that the errors of their relative positions could not be easily discoverable by the ship's chronometers; and they themselves must ultimately depend on astronomical observations, of which it is important to remark, the number necessary for an unimpugnable determination appears to be very great." A list of the Secondary Meridians selected by Raper, with the districts for which they are intended generally to serve, and their adopted longitudes, will be found in the "Practice of Navigation," p. 380.

"The method of surveying by chronometers alone, to the exclusion of astronomical observation," observes Raper in continuation, "has already been extensively adopted; as in Smyth's surveys in the Mediterranean, Owen's surveys on the coast of Africa, Fitzroy's in South America, &c. The principle advanced, therefore, is not new, but in the present state of hydrography it is important to urge the necessity for making astronomical determinations a totally separate consideration, and to suggest the advantage of a common recognition

* "Practice of Navigation," pp. 379, 380.

of fundamental points in the arrangement of chronometric differences."

Many concurrent circumstances of a favourable nature, and peculiar to the present time, seem to be conducive to the more systematic application of chronometers on board ship to the objects of science in the measurement of differences of longitude. The mechanical construction of chronometers has attained a high and unexampled degree of perfection, these improvements having at the same time been accompanied by a very considerable reduction in their cost; so that chronometers are now no longer rare instruments only within reach of the wealthy; and in lieu of one solitary chronometer as formerly, it is now not unusual to find three or more good chronometers on board every ship. The more general diffusion of a good practical education among young officers, both in the royal and mercantile navies, at the same time renders them more capable of applying chronometers to the accurate purposes of science and better able to appreciate their results. The increasing application of steam machinery, moreover, to men of war, whereby the average duration of passages will be much shortened and return voyages greatly facilitated, and the extension of lines of ocean steam navigation by the great mercantile companies to all parts of the globe, seem to afford, simultaneously with the above-mentioned circumstances, increased facilities for the systematic and careful measurement of chains of meridian distances.

If this view of this important question be correct, and the writer's partiality for a favourite subject has not caused him to take a too favourable view of present circumstances, it is important to consider whether the time has not arrived when it might be advisable to attempt to concentrate and condense in a practical form the necessary instructions for the guidance of those, who may be desirous of undertaking the accurate and systematic measurement of chronometric differences of longitude.

The information contained on this subject in the ordinary treatises on navigation is very meagre and unsatisfactory; not that this circumstance is to be imputed to them as a fault, for in truth the scientific deduction of chronometric differences forms no part of the ordinary processes of navigation, and information relating thereto might more justly be sought in a

work especially devoted to that subject. No such treatise, however, at present exists; many detached hints are extant in a fragmentary form, or interspersed in isolated papers among many books. The various volumes of the "Nautical Magazine," the appendices of several scientific voyages, some papers in the "Connaissance des Tems," and numerous pamphlets, may be quoted in illustration; and, in addition, some amount of floating knowledge exists in a traditional form as the results of experience, and is handed down from time to time, and from ship to ship, as one generation of officers is succeeded by another.

Speaking generally, we apprehend that, unless from peculiarly favourable circumstances of previous service in a surveying ship, on a scientific voyage, or under a scientific chief, officers in general have no organised knowledge of the minutiæ and details requisite to be attended to in the accurate measurement of meridian distances. No single book supplies the required information, and an officer furnished with some good chronometers, and desirous of advantageously employing them, has probably to grope his way as best he may, and devise a system for himself.

This want we propose to endeavour to remedy. With this object in view, in the following pages we shall endeavour to collect together and arrange various hints relating to the custody and management of chronometers on board ship, at present existing only in a traditional form, or to be found scattered amid many books often not easily accessible. The questions relating to errors and rates next engage attention; and subsequently, the formulæ for the determination of meridian distances are discussed and arranged in an organised and systematic manner, worthy, it is hoped, of the present advanced state of hydrographic science.

Amid much that cannot be new, and probably much that is familiar, the scientific reader may yet, we hope, find some fresh matter worthy of attention, and much that is old presented in a new or more instructive form. The utility of chronometric determinations, and the value of their results, as well as the possibility of their advantageous incorporation with the previous labours of others, much depends on the degree of care bestowed on the minute details of their manipulations. Many series of observations have often had their value materially impaired by

the accidental neglect of some small particulars; and doubtless a few measurements executed with a due regard to accuracy of detail, are many times more valuable than much larger masses of observation reduced and recorded in a loose, uncertain, and unsystematic manner.

Chronometric determinations obtained with no special regard to accuracy, reduced approximately, and recorded vaguely, although perhaps formerly valuable contributions to our then knowledge, are unsuited at present to the existing condition of maritime geography. "We are also to bear in mind," observes Raper, "that the ultimate perfection of hydrography demands very different proceedings from those which have sufficed to collect together the first rough materials of the outline; and," continues the same writer, "it can evidently only be effected by the chronometric measurement of small distances, finally depending on unimpugnable astronomical observations."*

As a contribution towards the improvement of geographical science, and in furtherance of the development of the above views, these pages have been undertaken: and if in the hands of scientific officers or intelligent travellers they are found at all conducive to the systematic realisation of this important subject, they will not have been written in vain, or the author's labour lost.

* "Naut. Mag." 1839, p. 320.

CHAPTER II.

RECEPTION AND STOWAGE OF CHRONOMETERS ON BOARD SHIP—EFFECTS OF MAGNETISM—EFFECTS OF TEMPERATURE—INFLUENCE OF OTHER CAUSES ON CHRONOMETERS—WINDING UP CHRONOMETERS—STANDARD CHRONOMETER—DISTINCTION OF CHRONOMETERS—COMPARISON OF CHRONOMETERS—CHRONOMETER JOURNAL, ETC.—ASSISTANT WATCH—MISCELLANEOUS OBSERVATIONS.

Reception and Stowage.—When chronometers are received on board ship, it is of importance that they should be at once stowed in the place prepared for their reception, in the position which it is intended they should permanently occupy; and when once suitably located, they should on no account be subjected to subsequent removal or displacement. The possible contingency of the ship being docked for extensive repairs, in which case their temporary removal would be unavoidable, is the sole exception to this general rule, excluding, of course, the accidental necessity for sending any particular chronometer on shore for the purposes of repair.

In selecting a place for their reception, much must of course depend on the size and accommodation of the ship, the particular nature of the service on which she is employed, and on the guidance of other circumstances; but, if possible, under favourable auspices, the following general conditions should be attended to:—

The chronometers should be placed low down in the ship (both because there is there less motion, and because the temperature is more equable); amidships; as far from the extremities, and as near the centre of motion, as convenient; not near the chain cables or other large masses of iron, so as to ensure freedom from the possible disturbance of magnetic influence; not in drawers, where the tremor caused by opening and shutting them acts injuriously on their balances, nor suspended from the deck

in cots or swinging tables, which has been proved by experience to be objectionable.*

The master's cabin in men-of-war is also objectionable, because there the chronometers are not *amidships*. So also the casing of the rudder-head in the captain's cabin, where we have known them to be placed, and where, doubtless, they looked very ornamental and scientific, is likewise to be deprecated, because they are there subjected to the more violent motion of the extremity of the ship, to the possible vibration of the rudder, and to the injurious influence of draughts of cold or damp air from the stern-windows: we believe, however, that this practice has now very properly been discontinued.

In vessels fitted with screw-propellers, the chronometers should be placed sufficiently far forward, as not to be affected by the vibration of the screw when under steam.

In Her Majesty's ships † a suitable place is now generally fitted for the reception of the chronometers in the after-cockpit or orlop-deck. In frigates, the after-part of the steerage, while unobjectionable in other respects, would appear to be both accessible and convenient. In flush-deck vessels the fore-part of the captain's cabin, amidships, would seem to be the most appropriate place; and in merchant-ships, the master's cabin, amidships, and as far forward as possible, perhaps offers the only eligible position.

* In the scientific voyage of the Chanticleer, under Capt. Forster, the chronometers were suspended in this manner. Tiarks is of opinion that the fact of several of them not retaining their rates for any length of time, and repeatedly altering them to a considerable amount in short intervals, is attributable to this cause; and he adds, that his own experience likewise proves, that suspension from the upper deck of a vessel is not favourable to the regular going of time-pieces. (Forster's "Voyage," vol. ii., Appendix, p. 226.) Capt. Owen remarks, that when suspended, they are liable to receive a vibratory motion that may affect their balance-wheel, besides being exposed to shocks; and, further, that the very necessity for taking them down, while comparing or winding up, is a sufficiently strong objection to the practice. ("Owen on Longitude," p. 6.) To this I may add, that the Rev. Mr. Fisher informs me that he has found, in some experiments made by him on land, the rates of chronometers sensibly affected by placing them on swinging tables; and if this be the case when perfectly quiescent in a room on shore, what might we not expect on board ship?

† It may be as well to state here, once for all, that the observations and precepts throughout these pages have been chiefly written with a view to the guidance of officers of H.M. Navy, and with regard to the arrangements of men-of-war. It is hoped, however, they may not be without their use to intelligent officers in the Mercantile Marine.

The best mode of stowing them seems to be as follows:—A box, divided into as many partitions as there are chronometers to be stowed in it, should be securely attached by screws to a solid block* of wood, about thirty inches in height, and firmly bolted to the beams of the deck below. Each partition should in depth be about equal to that of the largest box of the chronometers to be placed in it, and in length and breadth about two inches longer than the sides of the box it is intended to receive; the partitions, moreover, should be furnished either with separate lids, or there should be one general lid to close the whole box. Great care should be taken that the block and partitioned box thus prepared should be entirely detached from all contact with contiguous stanchions or bulkheads; and the block and box, moreover, should be surrounded with a strong external casing, the sides and lid of which should on no account be permitted to touch it, a clear space of at least two inches being left all round.

Previous to placing the boxes containing the chronometers

* During the voyage of the Beagle the chronometers were placed in box partitions, and packed in sawdust, as described above, "upon one of two wide shelves placed in a suitable position in the ship, low down, and as near the vessel's centre of motion as could be contrived."

Admiral Fitzroy states, that "thus placed, neither the running of the men upon deck, nor the firing of guns, nor the running out of chain-cables, caused the slightest vibration in the chronometers;" and he seems to give a preference to this mode of stowing over that of on a solid block; but as, after all, the chronometers can only be defended, do what you will, from the effects of vibration within the ship, and cannot be secured from those arising from the concussion of the ship herself, if she strike the ground, we do not see the occasion for this preference.

Capt. Owen recommends ("Owen on Longitude," p. 6,) as the most unobjectionable method, "that of having a table hung on gimbols, with a weight of from 20 lbs. to 50 lbs. suspended underneath, in the manner of an azimuth compass, so that the centre of gravity be as near the centre of its motion as possible, to permit it to keep its level permanently without being subject to vibrate, and the axes of the gimbols working in smooth stuffed-leather sockets bearing against springs in every direction, these springs being neither too stiff nor too sensible for the weights they are to support. The pillar or stand for the sockets to be a fixture in the deck. Such a contrivance would go far to prevent the ill effects of the ship's motion or concussion from firing guns, &c. The chronometers to be placed in small partitions on the table, and to be wedged securely in their places by soft cushions.

Should the above method not be resorted to, and it might not always be practicable, he further recommends in lieu "a table well secured to the deck, and kept unconnected with the adjacent bulkheads, the chronometers being placed and secured on its top as above directed."

Capt. Barnett adopted a table supported on a pivot like a compass-card; the jarring effect being relieved by a spring inside the pivot.

within the partitions appropriated for them, it will generally, we apprehend, be found convenient to unscrew and detach altogether their lids, because on account of the manner in which they are usually fitted;* if this be not done, when open they occupy much more room, and will require the partitioned spaces for their reception to be made inconveniently large. No detriment to the chronometers need be apprehended from this removal of their lids, as they will subsequently be sufficiently protected from the chance of injury by the covering lids which close the partitions.

Each chronometer in its box thus prepared, and moving freely in its gimbols, is then to be placed in the space allotted to it on a bed of horsehair, cotton, or shreds of bunting, about three inches thick,—the interstitial spaces around its sides being stuffed with the same material to within half an inch of the top of the inner box. Of the three substances named above, we prefer horsehair.†

Generally speaking, it is not the custom to receive the chronometers on board the ship until a few days before leaving the harbour, the time of the officers who will subsequently be charged with the duty of superintending their performances being at that period much occupied with other important matters connected with the equipment of the ship; but we think, that as from the influence of the new circumstances under which the chronometers are placed, the effects of motion in removing them, change of temperature, and possible action of magnetic causes, their rates after a while may differ from their previous rates on shore, it is very advisable, when practicable, that they should be received on board at an earlier period, so that they may become *naturalised* in their new position, and may have settled down to a stability of rate under their new conditions before the ship is called on to proceed to sea.

Effects of Magnetism.—Undoubted instances are on record of the performance of chronometers being affected by the action of

* Sometimes the upper part of a chronometer-box is fitted with a glass-plate, secured from injury by an additional hinged or sliding lid, so that the dial-plate may be viewed without opening the box; but this arrangement is not favourable to accuracy of comparison, on account of the reflexion and loss of light in viewing the dial-plate through two glasses, and from the possible error of parallax, which may deceive the eye.

† Coarse, dry sawdust has also frequently been used, but as it is liable to absorb moisture, and then to cake, harden, and lose its elasticity, it is not to be recommended.

magnetic influences, chiefly owing to the fact of the balances having accidentally acquired polarity.* Since attention has been called to this subject and greater caution exercised in the construction of the balances and balance-springs so as to avoid this source of error, instances of this kind are no doubt now extremely rare, and quite exceptional, in modern chronometers of good construction.† We should, however, be taking a very limited view of the subject, were we solely to confine our attention to the consideration of *permanent* polarity in chronometer balances, and lose sight of a possibly more prolific source of error, which, in all probability, is caused by the *induced* magnetism of considerable masses of iron on ship-board, in various stages of development, varying in different positions of the ship, and change of magnetic latitude.

Owing, no doubt, to the difficulty of arriving at definite conclusions on a matter so subtle and minute, as the influence of

* The earliest account we have of the effects of magnetism on chronometers seems to be that of Mr. Varley (reported in the "Philosophical Magazine," vol. i. 1798), who discovered that the balance acquired polarity at two opposite points on the rim, and thus that the going of the time-piece was affected *by the position of these poles with respect to the magnetic meridian.* Varley, moreover, found that every new balance which he tried was already more or less polarised.

In 1821, Professor Barlow, who does not appear to have been acquainted with Varley's previous inquiries, made a very complete series of experiments, showing that the vicinity of masses of *unmagnetised* iron invariably affected the rate of chronometers placed near them; no doubt from the accidental polarity of their balances.

Mr. Fisher also found ("Naut. Mag.," 1837, p. 160) that the rates of chronometers were sensibly affected by altering the position of the marks on the dial-plate with reference to the magnetic meridian.

Mr. Wackerbarth has recently pointed out a very curious instance of faulty performance in a chronometer, caused by the steel screws, with which the balance-wheel was loaded, having become sensibly magnetic. These screws being removed and replaced by others, the chronometer afterwards kept its rate very steadily. ("Monthly Notices, Royal Astronomical Society," vol. xix., p. 222.)

† In a letter to the Hydrographer to the Admiralty on the liability of chronometers to be disturbed by the earth's magnetism (Nov. 1857), Mr. Airy has recently stated that "of the *hundreds* of chronometers which have passed before me, and have been *regularly put under magnetic trial*, only *one*, viz. Buckland's, No. 425, has been sensibly affected by the earth's magnetism." An interesting paper by Mr. Airy relative to the performance of the chronometer alluded to, caused by the effects of the earth's magnetism on its balance, owing to its having accidentally become polarised, will be found in the "Nautical Magazine" for April 1840, p. 231, together with an explanation of the successful measures adopted by Mr. Airy to neutralise its effects.

Some interesting experiments "on the influence of the magnetic condition of ships on the rates of chronometers," have recently been made at the "Dépôt de la Marine," at Paris, by MM. Delamarche and Ploix, "Ingénieurs hydrographes."

magnetic causes, in which the phenomena produced are so liable to be masked by extraneous circumstances, considerable variety of opinion prevails, even among highly competent authorities;* however, inasmuch as, although probably in very rare and isolated cases, magnetic causes may possibly exert an injurious effect on the performance of chronometers, prudence, we think, requires us to recommend precautionary measures to obviate them. The following precautions are therefore suggested:—

The marks on the dial-plates of the chronometers should all occupy the same relative positions; that is, the line joining the twelfth and sixth hour-marks should all be parallel to one another, in order that, retaining the same invariable position with reference to the "fore-and-aft" line of the ship, they may be similarly affected by the possible local magnetic attraction of the ship's iron, and that in cases where the balances of the chronometers have accidentally acquired any degree of polarity, their mutual influences on each other may be reciprocal. Proximity

These experiments were made on nine chronometers, between June 1858 and January 1859, and were of the following nature. The chronometers were subjected to the influence of magnetized bars at different distances, and in different positions; the rates before, during, and after the trial, being very carefully ascertained, these magnets having previously, when similarly placed, with reference to a compass-needle, produced deflections from the magnetic meridian, varying from 15 to 40 degrees.

The variations of rate observed under these circumstances fluctuated from $0^{s}\cdot02$ to $0^{s}\cdot50$, the mean being only $0^{s}\cdot14$. MM. Delamarche and Ploix state, as their final conclusion, "The result of our experience is, then, contrary to the opinion entertained by some physicists, and seems to indicate that the magnetic condition of ships has no sensible influence on the rates of chronometers." ("Comptes Rendus des Séances de l'Académie des Sciences," tome xlviii. séance du 7 Mars, 1859.)

* Admiral Fitzroy suggested, as the possible cause of the discrepancy existing in the Beagle's chain of meridian distances, measured round the globe, which amounted to as much as 33 seconds, that chronometers may be affected by magnetic action in consequence of a ship's head being for a considerable time in any given direction.

Mr. Fisher thinks it probable that many discrepancies that have been observed by intelligent naval officers (which have generally been attributed to changes of temperature), have arisen from this cause.

On the other hand, Mr. Airy, in the communication to the Hydrographer before referred to (November, 1857), "On the magnitude of the magnetic forces in an iron ship," observes,—

"I have never been in any part of a ship in which a chronometer would be placed, in which the compass-needle was reversed, or in which it followed rudely the movements of the ship; there are places possessing this character, as very near the funnel, or very near some iron stanchions or knees, but nowhere where a chronometer would probably be placed. That is to say: The ship's magnetic force (practically) is never equal to the earth's magnetic force, and the earth's magnetic force will never be doubled by the addition of the ship's force. Therefore if the earth alone will not sensibly affect a chronometer, the earth and ship together will scarcely disturb it sensibly."

to vertical iron stanchions or knees, arm-stands of muskets or rifles, any large masses of unmagnetised iron, and in iron-built ships vertical iron bulk-heads, should also, in all cases, be very carefully avoided.

Effects of Temperature.—Ever since, in scientific voyages of modern times, the value of the chronometer has been recognised, as an instrument capable of being employed in the accurate determination of differences of longitude, changes of temperature have been universally admitted to be one of the principal causes of marked changes of rate. Great attention has in consequence been paid by the makers of these instruments to the mechanical details of their construction, to the improvement of their balances, and to the adjustment of their compensations. Considerable ingenuity has been employed, and with much success, in endeavours to eliminate or control the fluctuations of rate produced by variations of temperature.

The important effects which alterations of rate, caused by changes of temperature, would exercise on the measurement of meridian distances, seem to have engaged the attention of French navigators soon after chronometers were first employed for that purpose. Fleurieu, in his *Voyage de l'Isis*, in 1768, enters at length into this question, and proposes the employment of an "equation of temperature," to correct the irregularities of their rates. This matter is also alluded to in the accounts of the Voyages of M. Borda in the *Flore*, in 1772, and by M. Rossel in that of *D'Entrecasteaux*, in 1791.

This matter has also seriously engaged the attention of scientific officers in our own day. Admiral Fitzroy thus remarks: "During eight years' observation of the movements of chronometers, I have become gradually convinced that the ordinary motions of a ship, such as pitching and rolling moderately, do not affect tolerably good timekeepers, which are fixed in one place, and defended from vibration as well as concussion. Frequently employing chronometers in boats, and in very small vessels, has strengthened my conviction, that temperature is the chief, if not the only cause (usually speaking) of marked changes of rate. The balances of but few watches are so well compensated, as to be proof against a long continuance of higher or lower temperature. It often happens that the air in port, or near the land, is at a temperature very different from that over the open sea in the vicinity, and hence the difference sometimes found between

harbour and sea-rates. The changes so frequently noticed to take place in the rate of chronometers moved from the shore to the ship and the reverse, are well known to be caused partly by change of temperature, and partly by change of situation."*

"Some chronometrical measurements," continues the same authority, "have erred and caused much perplexity in the following manner. The chronometers were rated in air whose average temperature was, let us suppose for example, 70°. They were then carried through air, either considerably hotter, or considerably colder, and again rated in a temperature nearly equal to that specified. The rates were not found to differ much, and it was supposed the chronometers had been going extremely well, though, in truth, the rates of most of the watches had differed extremely (from those found in port) during the voyage; but they had returned nearly to their old rates upon reaching nearly equal temperature." †

The general tendency of alteration of temperature in an *uncompensated*‡ chronometer is, that an increase of temperature causes the rate to be retarded, or to alter in a losing direction;

* Notwithstanding their suspension in gimbols, chronometers are liable to a certain degree of oscillation, and alteration of horizontal position, when the ship is in motion. It has been suggested that this may affect their rates, by disturbing the regularity of the balance.

† "Voyages of Adventure and Beagle," Appendix, vol. ii., p. 326.

‡ MM. Delamarche and Ploix recently made some very interesting experiments on the effects of temperature on an *uncompensated* chronometer, placed at their disposal for that purpose by M. Bréguet, a French maker of celebrity.

The experiments were divided into two series, from May to September, and again during November and December, 1858, during which the chronometer was exposed to temperatures ranging from 0° to 35° Centigrade (32° to 95° Fahr.), while the corresponding rates were carefully observed.

The results showed a striking connexion between the rates and corresponding temperatures. At 0° the observed rate was + $3^{m}\ 33^{s}\cdot 5$, and at 35° it was — $2^{m}\ 52^{s}$. Between these two limits the rate constantly diminished as the temperature increased. The difference between these two extreme rates is $6^{m}\ 25^{s}\cdot 5$, which divided by 35 gives 11^{s} as the average change due to one degree of temperature centigrade (which corresponds to $6^{s}\cdot 11$ for a degree of Fahrenheit). Assuming, then, that the rate varied mathematically in proportion to the temperature, these gentlemen then formed a table giving the probable temperature corresponding to the observed rates, which was then compared with the observed temperatures. The column of differences in no case shows a greater variation than 1°·6. In 115 differences 77 are below half a degree, and in 7 only some tenths above 1°. When we consider that these results may be affected by slight errors in the mode of determining the mean daily temperature, viz. by taking a mean of the indications of a maximum and minimum thermometer, they are certainly very remarkable; in fact, as MM. Delamarche and

a decrease of temperature, on the contrary, accelerates the rates, or produces an alteration in a gaining direction. The object of the compensation is to correct this defect, and to produce uniformity of rate in spite of alteration of temperature. The artist has the power, by the adjustment of the compensation weights, to cause the chronometer to maintain the same rate at the two selected temperatures for which he purposes to regulate the compensation. At intermediate temperatures, and at temperatures outside the chosen limits on either side, the rates may be expected to vary; for the further correction of which "secondary or auxiliary compensations"* have been proposed. The smallness of

Ploix observe, "we are then justified in considering this uncompensated chronometer as a thermometer giving the mean temperature (n degrees) during the 24 hours by a daily rate equal to

$$+ 3^{m}\ 33^{s}\cdot 5 - n\ (11^{s})."$$

("Comptes Rendus des Séances de l'Académie des Sciences," tome xlviii., séance du 24 Jan. 1859).

The results of these experiments suggest some very important considerations. It is clear that in an uncompensated chronometer the corresponding variations of rate and temperature have a much more simple relation to one another than when the effects of the latter are modified by the compensation; no difficulty, therefore, would be experienced in arranging a series of tabulated rates corresponding to the different degrees of the thermometric scale. If, then, a chronometer with an uncompensated balance could be constructed with its variation of rate for temperature confined within very moderate limits, while at the same time careful means could be devised for estimating the mean daily temperature to which it was subjected, such an instrument would be as available for scientific uses, and probably as efficient, as the costly chronometers now in use. The time and trouble bestowed on the compensation is a large element in the cost of chronometers; uncompensated chronometers, being so much simplified in construction, might be sold at a greatly reduced price. By constructing the balance of gold or platinum, and no inner rim of steel being required, such a chronometer moreover would be entirely free from the possibility of magnetic influences.

* "When chronometers had been brought to great perfection, so as to go with scarcely any sensible variation of rate, while they were kept within moderate limits of temperature, it was observed that they always lost if the temperature either rose or fell beyond those limits; and on the other hand, if the compensation was adjusted for two extreme temperatures, then the watch always gained at mean ones. The cause of this was pointed out by the Rev. Geo. Fisher to the late Mr. E. J. Dent, and embodied by the latter in a Pamphlet on the "Errors of Chronometers," &c., which gave the following illustration of it. The diminution of force in the spring proceeds uniformly in proportion to the increase of heat, and may therefore be represented by a straight line inclined at some angle to another straight line which is divided into degrees of temperature. But the inertia of a compound balance cannot be made to decrease quite as fast as the heat increases; and therefore its rate of variation can only be represented by a curve, and can therefore only coincide with the straight line representing the variation of force in the spring, in two points, either the two extremes or two means, or one mean and one extreme point; or,

the variation, and the stability of the rate under varying temperature, is the test of the skill and success of the maker in the adjustment of his instrument. It sometimes happens that chronometers are "over-compensated," so that the effects alluded to above are reversed, the rate increasing with increase of temperature, and *vice versâ*. In consequence, however, of the difficulty of exactly regulating the compensation, it is to be understood, that these effects in practice are extremely uncertain, and hence it is impossible to predicate with any certainty what the effects on the rate caused by fluctuation of temperature in any particular chronometer may be.

If the size and arrangements of the ship permit, a separate cabin may be advantageously appropriated for the reception of the chronometers, a solid block and fittings, as already described, being placed within it. Such an arrangement affords the means, moreover, of adopting measures for maintaining in the chronometer-room "uniformity of temperature," which, under the circumstances already discussed, is a point of great importance. In the surveying voyage of Baron Roussin on the coast of Africa and Brazil in 1816–20 the temperature of the chronometer-box on board the Bayadere was maintained at a uniform standard (30° Centigrade, or 86° Fahr.) by means of a lamp; the admission of air being regulated by an aperture, the size of which could be adjusted at pleasure by means of a slide, the temperature of the interior air being shown by a thermometer. These precautions were rewarded by the watches performing their functions with extreme regularity.* In the surveying voyage, on the coast of Australia, of H.M.S. Fly, under the late Captain Blackwood, the temperature of the chronometer-room was in a similar manner maintained by a lamp at an equable standard, at times

in other words, the compensation can only be exact for some two temperatures for which you may choose to adjust it. The selected temperatures are generally 45° and 75°, 60° being the mean; for a greater range than 30°, the adjustment cannot be depended on.

"The correction of this error is called the 'secondary or auxiliary compensation,' and it is the point towards which nearly all recent inventions for the improvement of chronometers have been chiefly directed." (See "Denison on Clocks and Watches;" Weale's Series, p. 292; also "Dent on the Errors of Chronometers;" London, 1842.)

* "Recherches Chronométriques," p. 77. It has also been further remarked that artificially keeping the chronometers at a uniform temperature, likewise insures their being kept in dry air, and the maintenance of their oils in a state of fluidity.

when, from natural causes, it would otherwise have been liable to excessive variation.*

Another precaution was also adopted in the Fly to protect the chronometers from sudden draughts of cold air and to prevent the radiation of heat. Their box was covered over with a covering of coarse woollen (*fearnought*); over the partition containing the "standard"† chronometer, there was a flap in the cover, so that at any moment a comparison with the "standard" could be obtained without uncovering the rest. By this arrangement, under ordinary circumstances, it was only necessary to uncover the chronometer-box once a-day, for the purpose of winding and comparing; except in harbour, on those days when observations for the rates were made, when it was required to be done more frequently. As this device is certainly simple and easy of execution, we think it may safely be recommended for imitation.

Influence of other Causes on Chronometers.—Besides the possible effects of magnetism, and the undoubted effects of variations of temperature, experience shows that there are other causes in existence which exercise a very sensible influence on the performances of chronometers.

It has frequently been observed that many chronometers have a tendency towards a constant and progressive increase of rate, sometimes in a gaining, sometimes in a losing direction; but more frequently the former, altogether independent of the changes caused by variation of temperature. To this tendency has been given the name of the *acceleration;* great attention has been paid to its investigation by recent French writers, MM. du Cornulier and Lieussou,‡ to whose valuable labours nautical science is much indebted.

* It may be necessary to add a word of caution to those who may be disposed to adopt this device. The lamp and ventilating apparatus should be very carefully attended to, for if the temperature be allowed to become excessive, or the lamp incautiously permitted to go out for any time, the extreme variations of temperature to which the chronometers would thereby become liable, might probably be more injurious than if they were simply exposed to the natural changes of temperature of the atmosphere.

† See p. 26.

‡ The memoirs of M. du Cornulier, an officer in the French naval service, were originally published in the "Annales Maritimes" for 1831, 1832, 1842, and 1844; copious extracts from them have since been republished in the "Recherches Chronométriques," a most useful work, now in course of publication by the Depôt de la Marine at Paris, under the auspices of the Minister of Marine.

M. Aristide Lieussou, Ingénieur Hydrographe de la Marine, &c., is the author of a valuable work, "Recherches sur les Variations de la Marche des Pendules

Each of these gentlemen has independently* proposed algebraic formulæ for correcting the differences of longitude obtained by chronometers, which embrace not only the fluctuation produced by varying temperature, but also the effects of the progressive influence of the acceleration. In a subsequent part of this work we shall have occasion to refer more at length to the ingenious speculations and investigations of these authors; our only object here, is to introduce the matter generally to the notice of our readers, and to accompany it by a brief and popular explanation.

The "acceleration" may be defined to be the product of the mechanical imperfections which unavoidably exist even in the very best constructed chronometers, and is attributable to the effects of "wear, dirt, and the thickening of the oils (with which the pivots are lubricated), and which, in consequence, hinders or facilitates more and more the play of the moveable parts. Their effect is at first insensible; but as they are continually in action, the errors that they incessantly produce accumulate, and it becomes indispensable to take account of them after a certain lapse of time."†

In most watches the acceleration is a variable quantity, and presents no guarantee of permanent fixity;‡ we ought not, therefore, to assume it constant, but for small intervals; but, inasmuch as the performance of the chronometers can be checked, and their errors ascertained from time to time, by the necessary observations, "the hypothesis of the uniformity of the acceleration during moderate intervals," observes M. du Cornulier, "seems generally admissible." This uniformity is one of the principal conditions of the goodness of a chronometer. "In general," continues the same authority, "the acceleration seems to be so much

et des Chronomètres," Paris, 1854; which abounds with matter useful and instructive to those who take an interest in chronometric science.

* M. Lieussou states (note p. 30) that he was not aware of the previous labours of Du Cornulier, until he saw the report of the Consultative Committee of the Depôt de la Marine on his own memoir, in which the work of his predecessor was alluded to.

† "Recherches Chronométriques," p. 94.

‡ "An abrupt shock may alter the acceleration, because it may produce in an imperfect mechanism a new obstacle to movement, or destroy one which already exists. It is for this reason, that one ought to avoid putting a watch in a place where it would be exposed to any shock acting upon it directly or by transmission; a sharp pitching motion, a salvo of artillery, the proximity of a pump in movement, the fall of a heavy body near a watch, may suffice to develope the germ of a new acceleration." ("Rech. Chron." p. 96.)

the greater as the watch has been in movement for a less long time. It seems that these instruments require some months' exercise to settle down to a rate a little stable. The irregularities reappear after a long service, whether it be in consequence of wearing out, dirt, or the thickening of the oils; whatever may be the cause of the acceleration,* it is wise to bestow no confidence on watches which are affected by it to a considerable degree; it is always the sign of an essential vice, and often the prelude of a total derangement."†

Besides the progressive deterioration caused by wear, dirt, and the thickening or gradual evaporation of the oils, chronometers are sometimes injuriously affected by damp, which rusts the balance-spring and other parts composed of steel; when this happens, the effects are very destructive. To guard against it, some artists have lately adopted the precaution of hermetically closing the box which contains the works.

There is reason to believe that chronometers have sometimes been much affected by violent storms of thunder and lightning, not only when the ship has been struck by lightning, but even when the ship has only been in the midst of the storm. It is stated on good authority that such was the case in the voyage of the Coquille, under M. Duperrey in 1823. In the voyage from Amboyna to Port Jackson the frigate experienced some severe thunder-storms near the islands of Savu and Timor; when the vessel arrived at her destination the chronometers were found to have changed their rates enormously, from $-5{\cdot}3$, $-26{\cdot}2$, and $+10{\cdot}1$ respectively, to $+7{\cdot}0$, $-18{\cdot}7$, and $+27{\cdot}6$. The interval between the epochs of rating was 84 days,—a long period doubtless, and possibly some of the change was due to the effects of tem-

* "Recherches Chronométriques," p. 129.

† "We conceive," says M. du Cornulier, "that the tendency to advance (*i. e.* to go faster) results from a play more and more easy in all the parts of the mechanism, and especially of the balance upon its pivot; whatever may be the degree of exactness and of polish that the artist may have succeeded in giving to these pieces, the natural friction might still bring them to a more perfect degree of agreement, render their forms more regular, and annihilate the small asperities which had escaped him. Whilst this tendency is feeble we willingly recognise in it a perfection in the instrument itself."

"The tendency to retard (*i. e.* to go slower) seems to us to result from the progressive increase of rust, or from wear which injures the polish and becomes destructive to it; this is why we always suspect it. A degree of wear, which attains to a destructive limit, may equally give rise, according to circumstances, to an exaggerated tendency to advance." ("Rech. Chron." p. 127.)

perature—the chronometers being taken into colder latitudes and a lower temperature; but still the facts are very remarkable, and it would be unwise to deny the possible influence of the unwonted electric conditions of the atmosphere during these storms on their performances.*

Winding-up Chronometers.—The chronometers having been received on board, and stowed in the manner previously described in the place appropriated for their reception, it is of importance at once to commence and adopt a uniform and systematic manner of winding them up, and comparing them daily with one another.

Methodical arrangements in these particulars favour the stability of rate of the chronometers, assist in the detection of irregularities, and diminish the probabilities of their being accidentally allowed to run down;† while in the reduction of chronometric observations for the determination of meridian distances, systematic plans of comparison are indispensable to accuracy of computation, and very influential in diminishing the amount of labour required, if the number of chronometers employed is at all large.

It is of course a matter of purely arbitrary convenience what time is selected for the daily winding and comparison of the chronometers; the only point of importance is, that whatever arrangement be adopted, it be uniformly and systematically adhered to.

The practice of different scientific officers, of acknowledged repute, engaged on surveying duties, has varied considerably.

Capt. Owen states, that their chronometers were always wound and compared at noon; Capt. Fitzroy's plan was to wind up at 9 A.M., and compare at noon; while Sir Edward Belcher

* These facts are stated by M. Givry, "Ingénieur hydrographe de la marine," in a memoir "Sur l'emploi des Chronomètres à la mer," published in 1840, and which has been in great part reprinted in the "Recherches Chronométriques;" see p. 81.

† It is advisable to adopt, in the daily routine of the ship's duties, some precaution to prevent the possibility of the chronometers being accidentally omitted to be wound up, and to devise some plan by which this important duty should not be altogether dependent on the memory of any one single person. In many ships the cabin-door or steerage sentry reports both to the master and to the officer of the watch when the time has arrived for the performance of this duty; and he is subsequently not allowed to be relieved from his post until the serjeant or corporal of the guard has ascertained from the officer in charge of the chronometers that the operation has been performed, and has duly reported the same to the officer of the watch and to the commanding officer. Some arrangement of this nature is extremely necessary, and should always be adopted.

prefers 6 A.M. for both operations, thus leaving the whole of the day available for more active duties.

On the whole, we think 8 A.M. to be a time conveniently adapted to this duty, and consistent with the general arrangements of a man-of-war. The officers generally charged with this important function are not, under ordinary circumstances, likely to be then absent on detached duty, nor are they usually engaged in the customary morning evolutions. In cases, moreover, where it is proposed to take *sights* for time, during the forenoon, the usual morning comparison may be made subservient to that end, and taking the place of the first comparison before the observations, may save the necessity for an extra special comparison.

The chronometers should be wound *first* and compared *afterwards.* In winding them they should habitually be attended to, in the same order, from day to day, one by one, as they lie in their places; so that the mechanical habits of regularity in this particular may be a safeguard against the caprice of memory or accidental distraction from any disturbing cause. From want of system in this particular, we have known instances where the attention of the officer engaged in this duty being accidentally distracted during its performance, the chronometers have been compared, but some or all of them not wound up, that operation being forgotten, and the omission not detected till the chronometers ran down.

In winding up chronometers the turns of the key should always be counted, and the last turn made gently and carefully, *until it is felt to butt.* It is said, that it has sometimes happened to persons over careful, that they have let their chronometers run down by having counted the number of turns, and never winding up close for fear of injury to the chain or works, by which they have always lost a little of the chain each day; so that, after a time, the chronometer is found to stop just at the time it should be wound up.

In winding up box-chronometers, the chronometer should be inverted carefully in its gimbols, held firmly in the left hand, and the key pressed close home with the right; after the operation is performed, and the key withdrawn, care should be taken that the keyhole is again covered with the slider to secure it from dust or damp, and then the chronometer should be *gently eased down* into its natural position without violence or jerk.

Many chronometers are fitted with permanent keys* at their backs, which in some respects seem preferable to moveable ones, especially if they are fitted with rack-work to prevent the ill effects of attempting to turn them the wrong way.

Sir E. Belcher suggests the advantage of furnishing box-chronometers with the means of performing the operation of winding without disturbing their position, by adapting a spindle to their sides; such an arrangement could easily be effected, and would, possibly, be advantageous. Probably, also, the operation of winding in bad weather would be attended with less difficulty and risk.†

In winding up pocket-chronometers, the watch should be held firmly in the left hand with the wrist pressed gently against the breast, the key should be turned equably and steadily with the right, care being taken to avoid giving the watch any circular motion upon the key. The common but vicious practice of turning the left as well as the right hand is injurious, for two reasons: first, because the circular motion affects the regularity of the balance; and, secondly, because the compound motion of the two hands doubles the velocity of winding, and increases the chances of straining or snapping the spring from the jerk at the conclusion of the operation.

Chronometers and watches which are wound at the back usually require the key to be turned from right to left. Those having their keyholes on the dial-plate, on the contrary, from left to right.

The chronometers should be wound *daily*, whether constructed to go for one or two days; eight-day chronometers once a-week,—say on Sunday, a day easily remembered. If the number of chronometers is large, the precaution should be taken

* The chronometer-makers do not generally seem to be in favour of this arrangement. It is thought that dust and damp are more likely to penetrate through the fittings of the permanent key, than when the key-hole is closed in the usual manner by a spring slider.

† M. Givry states that "MM. Motel and Bréguet, two French watchmakers of celebrity, constructed chronometers which were wound up from the top of the box in which they were enclosed (how is not exactly stated), and that this ingenious innovation dispensed with the necessity of inverting their suspension."

On this, M. Delamarche, the editor of the "Recherches Chronométriques," remarks, "These artists have returned to the plan of winding up watches from the bottom, and consequently by reversing them; they say, that this movement is salutary on account of its distributing the oils over the friction points." ("Rech. Chron." p. 74.)

of examining them after winding, to see that none have been accidentally forgotten, either by looking at the winding index, if furnished with that apparatus, or by trying the key.

Standard Chronometer.—The chronometers having been all wound up, they should next be compared. To facilitate their systematic comparison, and to organise more easily the reduction of chronometric observations, it is customary to select one chronometer as a "*standard,*" to which all observations for the determination of the time are in the first instance referred; the indication of the other chronometers at the same moment being subsequently obtained by means of the comparisons. The chronometer selected to perform the duty of the "*standard,*" should be one of first-rate character, and by a maker of established repute: it will be convenient that it should have a clear and distinct beat to half-seconds; also, that its dial-plate be well marked; and it is advisable, although not indispensable, that its rate (at any rate at starting on the voyage) should be small. Stability of rate, however, is at all times much more important than the smallness of its amount. For the facility of comparison with the other chronometers it will be found convenient to arrange the chronometers in their box in such a manner that the standard shall occupy a central position among them. The XII and VI hour marks should all be parallel to one another, and to the "fore and aft" line of the ship.

Distinction of Chronometers.—As the description of chronometers by means of their makers' names and numbers is tedious and troublesome, it is often customary to denote each chronometer emphatically by a single letter, A, B, or C, &c. &c., an arrangement sufficiently distinctive to the persons who have to use them, and at the same time brief and concise when a series of results obtained from them has to be recorded; care being taken that in the commencement of the chronometric journal, and also in the records of their performances transmitted at any time to head-quarters, a preliminary notation is made of the actual maker's name and number belonging to each chronometer.

In accordance with this arrangement, it will be found convenient emphatically to denote the "standard chronometer" by the letter Z, the final letter of the alphabet, the other chronometers being represented by the initial letters, A, B, C, D, &c., in order of alphabetical sequence, or by the initial letter of their

makers' names, if preferred: such as A, for Arnold; D, for Dent; F, for Frodsham; M, for Murray, &c. &c.;* these little arrangements being, of course, matters of mere fancy and convenience. They are sanctioned, however, by the practice of high authorities; and having found them very convenient in our own experience, we think them worthy of being recommended for adoption, especially since, in addition to other advantages, as will be seen further on, this mode of description facilitates the algebraic discussion of the questions of errors and rates.

Comparison of Chronometers.—It is very desirable, when practicable, that the comparisons of the chronometers with the standard should all be made by one person. The effects of personal equation are thereby much simplified, and the chances of small contingent errors materially diminished.

The mode of proceeding is as follows: the observer, with book in hand, containing a ruled form for the entry of the time shown by the several chronometers takes a beat from the standard five seconds before the arrival of the second hand at any five or ten seconds mark, and then quickly casting his eye on the chronometer to be compared, counts with his ear the ten † beats which elapse before the second-hand arrives at the mark selected; at the completion of the interval he reads the other chronometer, and the comparison is effected. The operation being repeated a second time to correct the first judgment, the final observation is recorded. It will be found convenient to make the first comparison at 50^s, or at 20^s, and again at the minute or half minute for the final result.

This is done in succession for each chronometer. Half-a-minute elapsing between each comparison affords ample time to record the last one and to prepare for the next, while the fact of the standard being only noted at the minute and half-minute, makes the notation of its time a mere matter of form, and allows the writer's attention to be concentrated on the "*second*" of the other chronometer. When the round is completed, he takes the

* It will be found convenient to have the distinctive letter of each chronometer legibly printed on pieces of card-board, and conspicuously attached to their respective partitions in the chronometer-box.

† We are here supposing the "standard" to beat to half-seconds; if the beats are otherwise arranged, the observer must regulate his counting accordingly. Box-chronometers generally beat to half-seconds; pocket ones frequently beat five to two seconds.

precaution of seeing that the "*minute*" of the standard has been duly noted, so as to ensure that no constant error affects all the comparisons.

To compare chronometers, however, satisfactorily in this manner, requires much practice and self-confidence, and demands also a more delicate organisation of ear and eye than is possessed by all persons. The deficiency of natural or artificial light in the chronometer-room, moreover, and the difficulty of self-abstraction amid the babel of noises which sometimes prevails aboard ship, may render the assistance of a second person indispensable. The mode of comparison by two persons is as follows:—

The *principal*, who notes the time by the standard, also records the result in the ruled book previously prepared. The comparisons are made at the minute and half-minute. As the second-hand approaches 50^s or 20^s, the *principal* says, *Look out;* at 50^s or 20^s, *Stop;* sharply and audibly: but this is only a caution. At the minute or half-minute he repeats the word *Stop*. The *assistant* corrects his first impression by his second observation, and reports his reading to the *principal*, who writes it down. If necessary, an intermediate comparison, to assist the accurate observation of the subdivision of the second into tenths, may be made at the 55^s or 25^s, but this is rarely necessary to two persons accustomed to work together.

In stating the indication it is customary and convenient for the assistant to mention the seconds and tenths first, as most important; then the minutes; and, lastly, the hour, as of least consequence, since gross errors are always easy of detection: thus, the record of $7^h\ 54^m\ 23^s{\cdot}3$ is thus given: "twenty-three, three," "fifty-four," "seven;" the *principal* writing it down (as spoken) in reverse order.

If any mistake occur or confusion arise, whether the comparisons be made singly or with the aid of an assistant, it is advisable to pass that chronometer by for the moment, complete the comparisons of the others in regular order, and then return to the faulty one again.

The comparisons when completed will stand recorded in the chronometer-journal as in the first form in the following page.

If it be thought advisable to keep a fair copy of the comparisons, the second form will be found convenient.

Form No. 1.*—Chronometer Journal for record of daily comparisons of chronometers.

H.M.S.____________

Date.	Bar.	Max. and Min. Ther.	Chron. A.	2d Diff.	Chron. B.	2d Diff.	Chron. C.	2d Diff.	Initials of Comparers	Remarks.
1860. Sun. Jan. 1st.	in. 29·91	80° 78°	h m s Z 2 15 0 10 50 7 3 24 53		h m s 2 15 30 7 20 1 6 55 29		h m s 2 16 0 1 11 54 1 4 6		C.S. W.B.	Heavy Gale Much Motion
Mon. 2d.	in. 30·32	78° 71°	Z 2 39 0 11 14 5·3 3 24 54·7	s 1·7	2 39 30 7 43 48·8 6 55 41·2	s 12·2	2 40 0 1 35 55·5 1 4 4·5	s 1·5	C.S. G.C.	Confused Cross Sea

Form No. 2.*—Chronometer Journal for fair copy of comparisons, if thought necessary.

H.M.S.____________

Date.	Z — A	2d Diff.	Z — B	2d Diff.	Z — C	2d Diff.	Bar. and Mean Temp.	Remarks.
1860. Jan. Sun. 1	h m s 3 24 53		h m s 6 55 29		h m s 1 4 6		in. 29·91 79°	Heavy Gale Much Motion
Mon. 2	3 24 54·7	s 1·7	6 55 41·2	s 12·2	1 4 4·5	s 1·5	in. 30·32 74°	Confused Cross Sea
Tues. 3	3 24 55	0·3	6 55 52	10·8	1 4 3·7	0·8	in. 30·41 70°	Less Motion
Wed. 4	3 24 55·8	0·8	6 56 2	10·0	1 4 2·7	1·0	in. 30·34 73°	Exercised Firing at Night Quarters

It is advisable that the operation of winding and comparing should, as far as possible, always be performed by the same

* See also Forms, as amplified in the Appendix.

persons,* that these duties should be confided to as few people as possible; and in order that the errors arising from personal equation † should always lie in the same direction, that in making the comparison the parts performed by the principal and his assistants should not be interchanged. In order to guard, however, against the possibility of the persons usually charged with this duty being unable to do so from sickness or temporary absence, it is advisable that one or more persons should be instructed as occasional assistants; and it seems convenient that officers of the civil branch of the service should be chosen in preference, as less liable to be absent on detached duty.

Since, as has been previously observed, change of temperature is supposed generally to be the principal cause of marked changes of rate, it is desirable that the temperature of the chronometer-box be duly recorded, and accordingly means should be devised for placing within it a maximum and minimum thermometer, the indications of which, at the time of winding,‡ should be duly noted daily in the comparison book, so that if decided fluctuations of rate are observable in any of the chronometers, its cause may be sought among the records of the changes of temperature. The mean of the readings of the maximum and minimum thermometer is to be taken as the mean temperature for the preceding 24^{hrs}. The height of the barometer may also with propriety be noted at the same time.

As a matter of technical convenience, and as favouring uniformity of treatment, it is advisable that, for the purposes of comparison, the "standard" be always considered *fast* on the other chronometers, whatever their actual indications may be, 12^{hrs} being added to the time shown by the standard in taking their difference when necessary.

The utility of systematic comparison may be easily exempli-

* The initials of the persons who compare should be inserted in the comparison-book, in case a reference to them should be subsequently required to explain any seeming discrepancy.

† If all the observations were affected by a constant personal error, the errors of the chronometers on time only would be vitiated thereby, the rates which depend on the *difference* of errors not being influenced by them. So also, in measuring a meridian distance which depends on the comparison of two errors, its value would not be altered by a correction equally affecting both the times.

‡ The thermometers should be read off as soon as the box is opened, and before the admission of a fresh volume of air has had time to influence the indications of those instruments.

fied as follows:—If Z and A represent the standard and any other chronometer, we have from the records of the comparison-book the daily consecutive values of $Z-A$.

If the daily rates of the two chronometers happened to be the same, $Z-A$ would remain a constant quantity; if, as is more probable, the two chronometers had different rates, the consecutive values of $Z-A$, from day to day, would vary by the *algebraic difference* of their rates; if the rates remained uniform, this *second* difference would remain constant (disregarding, of course, small discrepancies arising from errors of comparison); if, on the contrary, the second difference were not uniform, it would show that the rate of one or both the chronometers was altering, and without further evidence it would be impossible to decide which was going astray.

If, however, we have more than two chronometers, a third one may be brought in to assist our judgment.

In this case, having three chronometers, Z, A, and B, our comparisons give us not only $Z-A$, but also $Z-B$, whence, without the trouble of direct comparison with each other, the differences of B and A may be obtained.

For $$(Z-A)-(Z-B)=B-A;$$
or $$(Z-B)-(Z-A)=A-B;$$

similarly for any other chronometer, C,

$$(Z-B)-(Z-C)=C-B;$$
or $$(Z-C)-(Z-B)=B-C;$$

and so on for any number of chronometers: in which processes all trace of the presence of Z vanishes.*

If, then, on examination of the records of the daily differences, $Z-A$ and $A-B$, it be observed that the second difference of the daily value of $A-B$ remained constant, while that of $Z-A$ is irregular, we may safely infer that

* This may appear, at first sight, a very circuitous and clumsy method of finding the differences of A and B, B and C, &c.; and so it would be if the number of chronometers were small, and these differences always required: but as the number of chronometers employed may be large, and as these differences are only required when there is a necessity for instituting a critical examination of the performances of the chronometers, perhaps at some subsequent period, this plan, we apprehend, will be found very convenient, and not liable to any just objection.

chronometers A and B are going steadily, and that Z is altering; if, on the other hand, we perceived that the second differences of Z — A and A — B were both irregular, while that of Z — B remained steady, it would indicate that A was the culprit, and Z and B retaining their rates, and so on.*

The system of the intercomparison of each chronometer separately with the standard, which we are here recommending, gives also great facilities for the manipulation of the results when a large number of chronometers is employed.

It is only necessary, in the first instance, to take into account the indications of the standard, as if it alone were concerned; the indications of the other chronometers are afterwards separately obtained by means of their respective comparisons.

Thus, Z being the time shown by the standard at the moment of any phenomenon, or at any given instant, the corresponding times indicated by the other chronometers are obtained at once by applying their comparisons, since

$$Z - (Z - A) = A$$
$$Z - (Z - B) = B$$
$$Z - (Z - C) = C$$

&c. &c.

That is, by *subtracting* the comparison for each chronometer from the time shown by the standard, we obtain the indications of the other chronometers in succession.

Thus, again, the primary result of any set of observations for the errors of the standard on local mean time, L, gives us

* In Her Majesty's Navy it was formerly customary to furnish every ship with one chronometer. If the captain supplied a private one in addition, then the Government gave another, so as to make three chronometers in all. It was argued that if a ship had but one chronometer, it would be unwise implicitly to trust it; and, therefore, great caution was necessary in navigation. If the ship had two, and they happened to differ, it would be impossible to tell which was right. If, however, she had three, the coincidence of any two of them would throw a strong probability on the truth of their results; while the mean of the three could probably be more safely relied on than any one of them taken singly: added to which, the examination of their intercomparisons, as pointed out above, gave the means of detecting which of the three was irregular.

By a recent more liberal regulation, the Admiralty now furnish all sea-going ships with three chronometers.

$Z-L$ if the error be considered *fast*, and $L-Z$ if, on the contrary, it be estimated as *slow*.

In the first case, for the errors of the other chronometers, we then have

$$(Z-L)-(Z-A)=A-L$$
$$(Z-L)-(Z-B)=B-L$$
$$(Z-L)-(Z-C)=C-L,$$
&c. &c.

That is, by *subtracting* the comparison for each chronometer from the error of the standard (adding 12 hours to the latter, when necessary), we obtain the error of that particular chronometer, and so on for the others in succession.

Again, if the chronometers are to be considered as *slow*, we have

$$(L-Z)+(Z-A)=L-A$$
$$(L-Z)+(Z-B)=L-B$$
$$(L-Z)+(Z-C)=L-C,$$
&c. &c.

That is, by *adding* the comparison for each chronometer to the error of the standard, we obtain the error of that particular chronometer (rejecting 12 hours when it exceeds 12 hours), and so on for the others in succession.

In the practical application of this system two points are essential: first, that, for the *purposes of comparison*, we agree to consider the standard fast of all the other chronometers, whatever their actual indications may be; and secondly, that whatever the times shown by the several chronometers actually may be (and, of course, during a long voyage they may exhibit considerable divergence, from the diversity of their rates), they may be treated in a uniform manner; that is, considered either to be all *fast* on local mean time, or else all *slow*. If this be done, we apprehend the management of any number of chronometers whatever will be a matter of comparative facility, and not nearly so formidable as might at first sight appear. It may also be useful to remark, that in all cases of uncertainty as to the proper application of the comparisons, the matter will generally be greatly simplified by expressing it algebraically, as in the preceding illustrations.

Chronometer Journal, &c.—A book (styled the "Chronometer Journal") should be kept for the daily record of the comparisons of the chronometers. The form No I., given in the Appendix (which is the same as the first one given at page 29), seems conveniently adapted for all practical purposes.* The book should be the size of foolscap or large letter-paper; it should be kept ruled up; the entries, in the first instance, should be in pencil; the daily differences of the several chronometers from the standard should be taken at the time of comparison, so that any error or discrepancy might be immediately detected, and a fresh comparison made, if necessary; the several entries should afterwards be inked in while the matter was still fresh on the memory; a wide column should be left at the right-hand margin for remarks, and notice should be taken therein of any circumstance likely to affect the performance of the chronometers, such as violent motion of the ship, her striking the ground, heavy firing of guns for salute or exercise, stoppage or removal of any of the chronometers, storms of thunder and lightning, &c. &c. The daily rates of the chronometers, determined from time to time by observation, should also be noted in the book, as a record, and for the purpose of occasional comparison with the column of second differences.

The chronometer journal, kept in the form and manner above described, should be solely appropriated to the daily record of the intercomparisons of the several chronometers with the standard at the time of winding. All other comparisons made at any other time for the purposes of observation for time, or for any other

* In the Admiralty instructions (for the government of Her Majesty's naval service), chap. v. p. 174, the captain is instructed to keep a chronometer journal, and a form of journal adapted for this purpose is given in the Appendix, p. 54. With every respect for Admiralty forms, we think those given in the Appendix to this work much to be preferred for all practical purposes. In the Admiralty form four columns are given for the comparison of each chronometer with the standard: the first two containing the times shown by the two chronometers at the moment of comparison; the third, their difference; and the fourth, their second difference. If this form be intended for daily practical use, it is very inconvenient to have two quantities, whose difference is to be taken, placed in parallel columns; and if intended for a permanent record, the first two columns are superfluous, as the actual times shown by the chronometers are a matter of no moment,—their differences and subsequent second differences being alone of any importance. Whatever view be adopted as to the propriety of keeping a fair copy of the comparisons with a view to a permanent record, one thing is quite clear, that in everything connected with these duties condensation should be studied, and all superfluities carefully avoided. We believe it is intended, shortly, to alter the form of these returns.

special object, should be made, on the principles previously recommended, in a separate memorandum-book set apart for that use.

If the chronometer journal be kept *neatly and carefully* in the manner above recommended, there does not seem to be any necessity for keeping a fair copy of it.* Copies involve a large additional amount of clerical labour. With every care, it is almost impossible wholly to avoid errors of transcription, especially in a matter involving a large mass of figures, in which the copyist can in most cases only take a mechanical interest; and, moreover, in case of doubt or dispute, the best-written copy has no value compared with the original.

The sole use of the chronometer journal is to enable the persons engaged in the management of the chronometers to detect irregularities, and to estimate the value of their results; and also to obtain, when necessary, by interpolation, their comparisons with the standard for any required moment when special comparisons have been omitted to be made.

Besides the "Chronometer Journal," it will also be found advisable to devote specially to the service of the chronometers, and to the computation of the meridian distances, a "Sight-Book," and a "Chronometer Work-Book." In the former are to be entered all observations of "single or equal altitudes" for the determination of time, errors, and rates, together with all occasional special comparisons and memoranda connected therewith. The latter should be exclusively appropriated to the calculations and reductions of the observations, for the determination of the errors and rates, and the computation of the meridian distances.

If fair copies be kept of the one, why not of the others? they are all equally essential and necessary; but, in truth, as Capt. Fitzroy observes, in recording observations "it would be of little use to give computations without comparisons, or comparisons without rates, or rates without the calculations on which they depend, or any part of these without the whole, which in any extended voyage would constitute a mass of figures filling several thick books."† The original books, therefore, containing the records of the comparisons, observations, and computations relating to the

* If, however, it be deemed necessary to keep a fair copy of the comparisons, the Journal form No. II. in the Appendix (which is the same as the second form at p. 29), is recommended for that purpose.

† Voyages of Adventure and Beagle, Appendix, vol. ii. p. 330.

chronometers, should be neatly and methodically kept, so that when their results are finally reported they may be available for inspection and reference if needed.

The "Chronometer Sight-Book," and "Work-Book," being as above recommended wholly and exclusively reserved for their own specific uses, all miscellaneous memoranda of common observations connected with the keeping of the ship's reckoning, or the calculations relative thereto, as well as generally all extraneous matter of any kind whatsoever, are to be rigidly excluded from them, and made in other books destined particularly for those uses. It may often happen, amid the multiplicity of affairs which engage an officer's attention on board ship, that the reduction of the observations relating to the chronometers cannot be made till a leisure opportunity offers, some time after they have been taken. It is then a great satisfaction to find all the necessary data clearly arranged, and recorded in such a manner that they may be confidently relied on.

Some of these remarks and recommendations may appear at first sight rather dogmatical and unnecessary; and, perhaps, only those who have been much engaged in operations of this nature can fully appreciate the advantages of methodical and systematic habits of comparison, observation, and computation, or can adequately realise the saving of time and labour involved in a careful attention to these minutiæ.

Assistant-Watch.—We have already established, as a fundamental maxim, that when the chronometers have been received on board and once stowed in their places, they should on no account be subsequently removed. The practice, therefore, of taking a chronometer on deck or on shore, for the purpose of observation, should never be resorted to; a pocket-chronometer, or a good pocket-watch with a second-hand, should on these occasions be used as an *assistant.** The actual time of any observed phenomena being then, in the first instance, taken by the assistant-watch; the corresponding times shown by the chronometers at the moment are subsequently obtained by applying the comparison.

* In the French naval service every vessel carrying a chronometer is, by a recent regulation, furnished also with a "*compteur*," for the purpose of the comparisons. ("Rech. Chron." p. 75.) A "*compteur*" is a small chronometer, fitted in a wooden box, without any gimbol suspension. They are used for "taking sights" with, and are supposed to go sufficiently well for that purpose. Their price is about 25*l.*

In comparing the assistant with the standard it will be found convenient, as in other cases, to consider the standard always fast: hence the comparison will give us Z — W, and W being the time shown by the watch,

$$W + (Z - W) = Z;$$

that is, the comparison being *added* to the time shown by the watch, gives the corresponding time shown by the standard.

As the assistant will rarely have exactly the same rate as the standard, the value of the comparison, Z — W, will be constantly varying; and hence, if the interval between the time of comparison and observation is at all large, the comparisons should be made both before and after, and their exact value at the moment of observation obtained by interpolation, and this should be done in all cases where observations are made on shore.

Thus, supposing the following comparisons to have been made:—

	Before.	After.
Z	$5^h\ 14^m\ 30^s$	$6^h\ 24^m\ 0^s$
W	3 11 16	4 20 48·7
Diff.	2 3 14	2 3 11·3

Also let the time by watch, corresponding to the observation on shore, be $3^h\ 56^m\ 19^s{\cdot}5$; here, in $1^h\ 10^m$, the comparison Z — W has varied by $2^s{\cdot}7$, and the interval between the time by watch at the first comparison and the observation is 45^m, whence we have

$$70^m : 45^m :: 2^s{\cdot}7 : x$$

$$\therefore x = \frac{45 \times 2{\cdot}7}{70}$$

$$= 1^s{\cdot}74$$

Consequently the comparison corresponding to the moment of observation will be $2^h\ 3^m\ 14^s - 1^s{\cdot}74 = 2^h\ 3^m\ 12^s{\cdot}26$, and the time of observation by the watch $3^h\ 56^m\ 19^s{\cdot}5 + 2^h\ 3^m\ 12^s{\cdot}26 = 5^h\ 59^m\ 31^s{\cdot}76$, the corresponding time by the standard.

The interpolation is made on the only admissible assumption, viz. that the variation of the comparison has taken place uniformly and in proportion to the time; and it may be computed either by common arithmetic or by the aid of proportional logarithms, as may be thought convenient.

It will be advisable, if possible, to appropriate a pocket-chro-

nometer to the purpose of performing the duty of assistant-watch; if a pocket-chronometer is not available, or cannot be spared, a good pocket-watch with a second-hand must be substituted: in either case, in order to give the watch so employed a fair chance of maintaining the stability of its rate, it should be kept in the chronometer-box, except when actually in use, and subjected to the same careful usage and delicacy of treatment as the other chronometers. Pocket-chronometers, or good watches employed as such, notwithstanding their name, should *never* be worn in the pocket. When carried in the pocket, they are placed in a vertical position, and subjected to the warmth arising from contact with the body. When removed from the pocket for use, they are held in a horizontal position, and probably subjected to a considerable alteration of temperature. The alteration of position is liable to disturb the regularity of the balance, and the variation of temperature to produce fluctuations of rate. A chronometer is merely a machine, after all; and a perfect chronometer, like a perfect circle or a perfect straight line, only a happy mathematical accident; and it is therefore unreasonable to suppose that it can be expected to perform its functions accurately, unless delicately treated and subjected to uniformity of physical conditions.

It will be advisable, therefore, to keep the assistant-watch in a small box, which, when carried on shore or on deck for the purpose of observation, should be maintained in a horizontal position, and, in its passage to and fro, should always be carried by hand. Care should also be taken that, when the box is open for the purpose of noting time, the watch should be kept in the shade, and not exposed to the direct action of the sun's rays, which, especially in a tropical climate, is likely to be very injurious.

Miscellaneous Observations.—1. When it may be necessary to move box-chronometers from the shore to the ship, or on other occasions,* they should be *locked* in their gimbols, so as to prevent an unusual oscillatory motion and to retain their hori-

* The Admiralty have recently issued the following instructions for sending home defective chronometers from abroad, or from the outports, to the Royal Observatory at Greenwich:—

"Defective chronometers being very liable to injury on their passage from abroad, or when travelling on shore, the following precautions in sending them are to be adopted:—

zontal position, under which condition alone their balances act with perfect fairness. In carrying them by hand, the best way is to sling them in a kerchief, passing the bight under the bottom of the box and the ends through the handles, tying them at the top in a reef-knot; care should be taken not to give the chronometers a vibrating or oscillating motion, and to carry them as steadily as possible. When conveyed in a boat, they should be held suspended by the hand.*

It has been sometimes customary, when salutes have been

"I. Wedge the balance with cork, so as to prevent any movement in it whatever.

"II. Turn the screw-catch into the gimbol-ring and the metal-box of the chronometer, and screw it there fast, so as to prevent either of them from moving. If there appears to be the slightest probability that the gimbolling can come loose, or the gimbol-block can fail, secure them by tying the screw-catch fast, putting a wood screw into the block, or adopting any other suitable measures.

"III. Fill the whole space between the metal-box and its wooden case with any stuffing, such as dry oakum free from dust, or dry paper shavings free from dust (wood shavings are sometimes objectionable, as bringing turpentine and dirt), or any other clean and soft material, so as to prevent the possibility of movement of the chronometer.

"IV. Having closed the wooden chronometer-box, it is to be placed in a wicker basket or hamper, or in a box of partially yielding character, and is to be packed in it with abundance of soft stuffing. If there is no wicker basket at hand, or nothing except a wooden box, this box must be surrounded with stuffing, and inclosed in canvas, so that it can never receive a jarring blow.

"V. Two or more chronometers (secured from injury in their boxes by the foregoing measures I. to III.) may be packed in a yielding case or basket, but all contact between them must be prevented by precautions similar to the above with straw or some stuffing material.

"VI. Address the package to the Astronomer Royal, in characters distinctly legible, adding the words 'With care,' the date, and name of the ship sent from, and, when by land, that of the rail by which it goes to Greenwich; a mail or passenger train being adopted, but never a luggage train.

"Admiralty Hydrographic Office,
December 1859."

* The difference of longitude between Greenwich and Valentia was recently measured with great accuracy by means of chronometers, under the immediate superintendence of Mr. Airy, the Astronomer Royal. Thirty pocket-chronometers were employed on the occasion, it being considered that they were less susceptible of injury by travelling than box-chronometers. The mode of carrying them was as follows: they were carefully packed in two cases divided each into fifteen compartments, with springs under each chronometer, pressing it upwards firmly, but gently, against a padded lid. The sides and tops of each case were well wadded outside to protect them from any violence or jar. A number of boxes was then made, each of which would just hold the two cases, placed one above the other; and to every railway-carriage, steam-boat, or mail-coach to be used in the course of their transmission to and fro between the several stations, one of these boxes was screwed down.

fired on board ship, or the guns fired for exercise, to remove the chronometers from their places, and hold them by hand; but if the chronometers are stowed in horsehair on a solid block, as previously recommended in these pages, we believe this precaution to be entirely unnecessary and superfluous, and likely to be rather detrimental than otherwise.

2. It would be advantageous, when chronometers are delivered from the dépôt to any ship, if information were furnished not only of the errors and rates, by the latest observations, as is at present customary, but also of *the age of the oils* (that is, the date when they were last renewed by the maker, when the chronometer was cleaned or repaired), and *the manner in which the chronometers are affected by temperature*, as is the custom in the French naval service.* The same remarks are equally applicable to chronometers supplied for the use of merchant shipping.

The age of the oils is very important,† and a knowledge of it will often account for a deterioration in the performance of a chronometer. The manner in which changes of temperature affect the performance of these instruments is, perhaps, even more so, and an acquaintance with it would often reconcile apparent anomalies otherwise inexplicable; as, for instance, when two chronometers which happen to be differently or even very unequally acted on by change of temperature, diverge in their indications.

3. The acoustic properties of tubes are well known, and the convenience of gutta-percha tubing has already caused it to be extensively introduced on board ship, for the purpose of conveying with facility messages from the deck to the engine-room and other places. We think it is well worthy of consideration, whether gutta-percha tubes might not also be applied with great

* In the French naval service, when chronometers are returned to the dépôt, a return is also forwarded giving a statement, 1st, of all the observed errors and rates referred to noon, mean time at the place of observation; 2d, the mean of the temperatures at 9 A.M. (the time of winding) of the chronometer chest, in the intervals of the observations; 3d, a summary indication of the circumstances of the voyage which have seemed to alter the rates of the watches. "Rech. Chron." p. 14.

† To show the importance attached to the age of the oils by the French hydrographers, we may mention that by the regulations of the Ministry of Marine, chronometers are permitted to be returned to the naval observatories, when the age of the oils exceeds three years, or when the vessel is about to proceed to a destination such, that there is reason to presume, that the age of the oils will exceed three years, before the end of the voyage. "Rech. Chron." p. 15.

advantage for the purpose of conveying signals to the chronometer-room. In taking observations at sea, the assistant-watch could thus be altogether dispensed with, and the chances of contingent errors arising from its use would thus disappear; no difficulty would probably be experienced in attaching the mouthpiece to the mouth of the observer (both his hands being engaged with his sextant), as the attachment could be made by an elastic India-rubber band passing round the back of his neck; and the tubing, being perfectly flexible, might be accommodated to his position without inconvenience. In noting the times of extraneous phenomena, signals from other ships, flashes of guns, bursting of rockets, &c. &c., its use would be equally applicable; and, doubtless, in bad weather at sea it would be of great advantage in common observations. The transmission of sound is practically instantaneous for short distances, and the advantage of dispensing, if possible, with the assistance of an intermediate watch very great.

4. If a chronometer has run down, it may be set a-going, after being again wound up, by giving it a small but moderately quick horizontal circular motion. In the absence of any direct evidence to the contrary, it may be assumed that the chronometer has resumed its former rate; but this is by no means certain, as it is frequently found to differ.

5. It has been found that chronometers generally perform best at the commencement of a voyage; as time advances since they left their makers' hands, their performances fall off, their results are less regular, and their rates generally accelerate.* These

* A table will be found in the Appendix, exhibiting the performances of the chronometers of H.M.S. Fly, employed surveying on the coasts of Australia from 1842 to 1846. The table exhibits the results of the experience of four years, during which the chronometers were subjected to great variety of climate, and to all the vicissitudes of a long voyage. The conclusions to be deduced from it are highly instructive, and tend to show that the general, though not invariable, tendency of all chronometers is to accelerate their rates as time advances since they were last adjusted by their makers. It also shows that, although after the lapse of a long period since the commencement of the voyage, the rates may exhibit considerable divergence from the values they had at starting, yet that these changes for the most part are gradual and progressive; and amid all their minute oscillations of amount, there is a much greater approximation to stability of condition, at any given period, than on a casual inspection might at first sight appear. This consideration tends to increase the confidence which may be placed in the performances of good chronometers during long voyages, and seems to show that they may be more fully relied on than is generally supposed.

A similar table will be found in the "Nautical Magazine" for April 1843, p. 226,

changes, no doubt, are due to the general mechanical imperfection to which their construction is liable, to the thickening of the oil with which their pivots are lubricated, and to the effects of friction, dust, and damp; for these reasons it seems advisable, in regulating the arrangement of the work of surveys, to measure the principal chronometric runs, if possible, at the commencement of the expedition, while the chronometers are in good order, their rates regular, and performances unexceptionable: advantage has been taken of these considerations in some recent surveys.

Stability of rate, as we have elsewhere remarked, is of much more importance than the smallness of its amount: chronometers with small rates are, no doubt, more convenient to work with than those whose rates are very large; but we have known chronometers with very large rates, say 40^s or 50^s a-day, give most unexceptionable results in the measurement of meridian distances: they are, therefore, by no means to be despised or rejected.*

giving the rates of some chronometers employed by Captain Barnett, during a period of three years in the West Indies. The general results are very remarkable, and show that, in this case, those which had originally a gaining rate, increased their rate of gaining; while those with an original losing rate decreased it, and gradually acquired a gaining one: that is, the tendency of the acceleration in all cases seems to have been towards a gradual increase of gaining rate.

The chronometers which have passed under our notice in other ships have usually exhibited the same tendency; and even in the best-constructed chronometers a gradual acceleration of rate may, we believe, be generally anticipated.

* We once had experience of an eight-day chronometer, whose rate was large and irregular. Its performance was much improved by adopting the practice of winding it daily, instead of once a-week.

CHAPTER III.

ON THE DETERMINATION OF TIME — TRANSIT OBSERVATIONS — SEXTANT OBSERVATIONS — ARTIFICIAL HORIZON — SEXTANT — METHODS OF OBSERVATION — EQUAL ALTITUDES — SINGLE ALTITUDES — COMPARISONS WITH THE STANDARD — POSITION OF THE PLACE OF OBSERVATION — REDUCTION OF THE OBSERVATIONS — PRECONCERTED SIGNALS.

Before we proceed to discuss the important questions relating to the determination of the errors and rates of chronometers, it will be proper to make some brief and practical observations on the modes of determining the time correctly, which are available to officers serving on board ship, and within compass of the means ordinarily at their disposal.

The observations which may be employed are of three kinds:—

I. Transit observations.
II. Those requiring the use of a sextant.
III. Preconcerted signals.

I. *Transit Observations.*—The first class of observations may be briefly dismissed with but few remarks. Ships employed on scientific voyages and on surveying duties have occasionally been furnished with portable transit instruments; of course, they will very rarely be found among the apparatus employed on board ships engaged in the ordinary duties of the service. Even among scientific officers of acknowledged ability and experience, the use of the portable transit does not seem to have met with much favour or approbation.

"The correct adjustment of a transit instrument," says Capt. Owen,* "for nice astronomical observations, is a delicate and tedious operation; and not by any means so simple as is generally

* Owen on Longitude, p. 11.

supposed: it is, therefore, not an instrument likely to be much in use among navigators, as it requires not only very strict attention to preserve it in adjustment, but the adjustment itself requires more time than can generally be given to it, either in men-of-war or merchant-vessels. It will likewise frequently be difficult to find a convenient place to fix it in for the period necessary to make it useful." And again, Capt. Fitzroy* observes, that "a transit instrument requires what is not always at command, viz. time; to be well placed; skill in its use; and habits of observing: which are neither readily nor easily acquired." And in the measurements of his extensive series of Meridian Distances, the time was always obtained by equal altitudes observed with a sextant.

Our own experience in this matter tends in the same direction. Satisfactory observations with a transit instrument require much time and many conveniencies not always easily attainable under the ordinary conditions of service at sea. Several requisites are indispensable; a convenient situation,—commanding a clear view of the heavens, north and south; a firm and stable support for the instrument, so as to ensure freedom from accidental tremor; a tent, or portable observatory of some kind, to afford shelter to the instrument; time, opportunity, and a favourable turn of fine weather. Unless these requisites are likely to be obtained, it is scarcely worth while to set up a transit instrument, or to take the trouble of forming a temporary establishment on shore, which it necessarily involves.

Small transit instruments, moreover, are very difficult to maintain in strict adjustment; the standards, or piers, being made of metal, are constantly altering their state of expansion, from the varying effects of temperature which, in a small and temporary observatory (more especially in a tropical climate), it is impossible to maintain in a state at all approaching to uniformity; hence the level error is constantly varying; and, let the adjustments be effected with what care they may, it is impossible to depend on their stability even for a short period. For these, and other obvious reasons, officers in general give the preference to sextant observations, which, when undertaken by a skilful observer, give the time with an accuracy but little, if at all, inferior to that attainable by transit observations: persons, however, who may be

* Voyages of Adventure and Beagle, Appendix, vol. ii. p. 328.

furnished with portable transit instruments, and disposed to use them, will find ample directions for the necessary adjustments and manipulations in many works devoted to such subjects.*

II. *Sextant Observations.*—The simplest and most convenient kind of astronomical observations available for the determination of time are those which can be made with a sextant and artificial horizon. In any observations having any pretensions to scientific accuracy, it is always to be assumed, as a matter of course, that they have been made on shore with the artificial horizon; observations made with the sea horizon can only be considered as approximations, and would only be admissible as data in the discussion of meridian distances, in cases where, from any peculiar circumstances, it had been impossible to obtain observations on shore. Altitudes observed with the sea horizon, even under the most favourable circumstances, are liable to be vitiated by three causes: first, the inaccuracy of the contact; secondly, the uncertain effects of refraction on the appearance of the horizon; and thirdly, the errors arising from an inaccurate estimation of the correction for dip.

In observations made with a mercurial horizon on shore, the contact of the limbs of the sun or of the images of a star is susceptible of being made with great precision, while the second and third sources of error vanish altogether; and, moreover, the effects of any error in the estimation of the index correction of the instrument, as well as those of observation, are halved in the resulting altitudes: while those proceeding from small errors in the assumed value of the sun's semi-diameter may be eliminated by observing both limbs.

Artificial Horizon.—Of the various kinds of artificial horizons that have been proposed, the common mercurial horizon is, without doubt, the best. It consists of a shallow trough filled with mercury, and screened from the wind, when necessary, by a glass roof. In using it, care should be taken to select a convenient place, where it will not be subjected to accidental tremors; rocky soil is the best, if attainable; sandy soil, or made ground of any kind, being very susceptible of tremulous motion. When the mercury is poured out, the forefinger being placed on the mouth of the bottle, and the bottle having been

* See Galbraith's Tables: Simms on Mathematical Instruments; "Penny Cyclopædia," article "Transit;" Pearson's "Practical Astronomy," &c. &c.

well shaken, it should be inverted, so that all the scum and dirt may rise to the surface, and the pure mercury alone pass into the trough. Care should be taken completely to fill the trough, so that the capillary attraction of the mercury to the sides may not interfere with its assuming a perfectly horizontal surface; and at the same time not to drain the bottle, so that no accidental impurities may be poured into the trough. If this happens, it is often very difficult to clear the surface; the best way of doing it, is with the edge of a silk handkerchief passed along it, but it is often easier to pour the mercury back into the bottle again, and pour it out again *de novo*.

If from the admixture of foreign dirt, or from the accumulation of its oxydation, the mercury become very foul, it should be cleaned from time to time by straining it through a piece of wash-leather. Care should be taken that the leather so used should not afterwards be employed to clean a sextant with, as the minute particles of mercury remaining in it, will form an amalgam with the silver of the arc and vernier, and may do them great injury.

The glass roof should be placed fairly over the trough, so that the rays of light from the object observed may pass through it in a vertical plane, and not obliquely, so as to avoid as much as possible the errors arising from any imperfection in, or non-parallelism of, the surface of the glass plates. The ends of the roof should be marked, and it should always be placed over the trough in the same invariable position, so that any errors arising from its use may be constants equally affecting all the observations. Many careful observers, however, are in the habit of reversing the roof during their observations; half being made with the roof in one position, and half in the other, the errors become neutralised. When the air is sufficiently calm, it is advisable to dispense with the roof altogether.

Sextant.—It seems desirable, when otherwise convenient, that the sextant devoted to observations for time on shore should not be indifferently hacked about for other purposes; it is of great importance that an observer should be able to place reliance on the trustworthiness of his instrument, and on the stability of its index error. If, therefore, he has the command of more than one instrument, the best may be suitably reserved

for the finer kinds of observation, such as lunars at sea, and determining the latitude and time on shore; and the other one to the common observations required in the ordinary navigation of the ship at sea.

The index error of the instrument should be frequently examined, and its value determined from time to time; ample instructions on this head will be found in the several treatises on Navigation. The actual amount of the index correction is a point of no importance, provided its stability can be depended on. The constancy of the value of this error is a test of a good instrument. In an instrument supposed to be good, no attempt should ever be made to eliminate the error by adjusting the screws; nothing tends to injure sextants more than tampering with the instrumental adjustments which have been made by the maker. From the alternate expansion and contraction of the metal from the effects of temperature, the values of the instrumental errors are probably in a state of constant minute fluctuation. These may be allowed for by a careful determination of the index error from time to time; but it is impossible ever wholly to remove them by mechanical adjustments.*

Observations with a sextant are liable to minute sources of error, from the imperfections of the shades or dark glasses. The sides of the coloured glasses are not always parallel, and, consequently, the divergence of the rays of light in passing through them vitiates the value of the angles measured. Again, glasses of different shades give different values of angles measured, and apparently different results for the value of the index errors. This chiefly arises from optical causes affecting the appreciation of the contacts. On this account it has been recommended to employ a dark glass at the eye end of the telescope instead. If this glass is not perfect, the rays from the object and the image are affected alike, and the angle between them remains unchanged. In cloudy weather, however, when

* "It is better that error should exist (observes Mr. Raper), provided that it is allowed for nearly, than that mischief should ensue to the instrument from ignorant attempts at a perfect adjustment; and the skilful observer, instead of implicitly depending upon the supposed perfection of his instrument, will endeavour to avail himself of those cases in which errors, if they exist, will destroy each other."—("Practice of Navigation," p. 165; which see, also, for other important remarks on the sextant.)

the brilliancy of the real and reflected images in the horizon is constantly varying, the use of the eye-piece is not practicable, and the coloured shades must be used instead. In this case neutral-tint shades seem far preferable to the common red, yellow, and green ones: they are more natural, less fatiguing to the eye, and, by permitting the two images to be adjusted to the same shade of colour, more favourable to freedom from optical defects of observation.

Time and trouble are often saved by having the tubes marked to the observer's focus.* Previous to observing, the precaution should be taken of seeing that there is plenty of drift for the working of the tangent-screw; after a set of observations has commenced, it is very unsatisfactory to have it interrupted by finding that the tangent-screw is choked,—an annoyance that a little previous foresight would have avoided.

Care should be taken, especially in tropical climates, that the sextant is not unnecessarily exposed to the action of the sun's rays, and, when not actually in use during the pauses between the observations, it should be kept in the shade, or screened with a handkerchief: nothing tends to ruin sextants more, than unnecessarily roasting them in the sun.

The degree of accuracy and satisfaction with which observations can be made with the artificial horizon depends much on the bodily health and condition of the observer. A clear head, a steady hand, a sharp eye, and a mind at once calm in its general disposition and acute in its perceptions, are elements in the success of the perfect observer. Hence, previous to observing, everything should be avoided which tends to agitate the nervous system, or to increase the action of the heart; such as violent exercise, or running: so, also, carrying the instruments to the spot of observation should be avoided, and performed by deputy, as carrying heavy weights with the arm in a constrained position tends to produce muscular tremor very unfavourable to the duty to be performed. The eye also should

* It would be extremely convenient if the fittings of sextant-boxes permitted the tubes to be put away in them, adjusted to focus; time is often lost in doing so; at night it is often very troublesome; moreover, the constant habit of pulling the eye-tube in and out frequently makes it work slack. If sextant-boxes were made square, as subsequently recommended, there would be plenty of room in them to permit this.

not be unduly fatigued previous to observing, and should not be unnecessarily exposed to glare.

Methods of Observation.—The modes of observation for the determination of time usual in practice, are either "equal or corresponding altitudes," or "single or independent altitudes." The objects observed may either be the sun or stars. The former is the most convenient, and generally preferred; the latter have some advantages, in a mathematical point of view, which, however, are perhaps neutralised in practice by the greater demand on the skill of the observer, and by the material inconveniences which attend night observations; we shall, therefore, here suppose generally the object observed to be the sun.

When the time is determined by single altitudes, the result is directly dependent on the exact correctness of the elements employed—the latitude of the place—the declination of the object observed, and its altitude. The former may not be correctly known; the second depends on the assumed perfection of the tables employed, and on the right estimation of the approximate longitude; the last on the skill of the observer, the correctness of his instruments, and on the true allowance for the value of the refraction. From the combination of these small causes discrepancies often arise in the results for time, especially when the observations are made on opposite sides of the meridian, which are very difficult to reconcile. Equal altitude observations, on the other hand, are in a great measure independent of these sources of error. If the object observed is a star, a knowledge of the true time flows at once from the observation without subsequent reduction; and if the object observed be the sun, as is more generally the case, the "equation of equal altitudes" required on the occasion is but slightly dependent on an exact knowledge of the latitude of the observer, or of the sun's declination. The value of the method of equal altitudes consists in the same altitude being noted on the opposite side of the meridian, without regard to the precise measure of it. Hence, as Raper observes, "the moving of the index destroys the integrity of the method, since the second altitude is no longer identical with the first, but is merely inferred to be equal to it from the reading.* The errors, however, are

* It would be a great advantage if the mathematical-instrument makers would abandon the absurd shape in which sextant-boxes are usually constructed, and make

greatly diminished by taking numerous altitudes; or a number of instruments may be employed, set to different altitudes." So, also, the theory of the method supposes that the value of the refraction remains the same at the period of the two observations; and thus that equal *apparent* altitudes actually correspond to the same *true* altitudes on each side of the meridian. No doubt, in ordinary practice, this assumption does not involve any appreciable error. In cases, however, where there is reason to suppose a considerable variation in the amount of the refraction, it has been proposed to allow a correction for it, and a formula for that purpose is given in the "Memoirs of the Astronomical Society," in a paper by Mr. Riddle, vol. iii. p. 216.

Equal Altitudes.—A common mode of taking equal-altitude observations is to set the instrument at any even division, a little in advance of the sun's double altitude at the moment, when viewed in the artificial horizon; the images of the sun are then overlapping, and gradually uncovering. At the moment of separation of the limbs the time by watch is noted by an *assistant* on a signal from the *principal.** The index of the sextant is then advanced through 10′ (or one division of a degree), and the operation repeated. This is done in succession through from five to ten observations,† so that in taking a mean, small errors may be eliminated. In the afternoon the observations are repeated in reverse order, the sextant being set to the same divisions as in the forenoon, beginning with the last observed. If any of the observations are lost, the corresponding one in the forenoon must be rejected.

The corresponding times of the forenoon and afternoon observations are then to be added together, and their sums meaned;

them rectangular, and so arranged that the sextant might be put away in them, with the index clamped at any part of the arc: for the convenience of night observations, when it may not always be practicable to read off at the moment, and in taking equal altitudes, when the last observation in the forenoon is the first in the afternoon, this facility would be found very useful.

* Skilful observers are sometimes in the habit of taking the time for themselves; no doubt there is great advantage in dispensing with the assistance of a second person when it can be done, but it requires much practice and habits of self-possession; and, moreover, the watch employed must have a very clear and distinct beat.

† Our own practice has frequently been to observe through two degrees: this gives thirteen observations (the sextant being cut to 10″), and, after retaining ten, allows an overplus of three for faulty contacts, or for the loss of duplicates in the afternoon.

the half of which mean result will be the mean approximate time by watch at the moment of apparent noon.

The observations will stand recorded in the following form:—

Altitudes.	Times, A.M.	Times, P.M.	Sums.
° ′	h m s	h m s	h m s
65 0	3 25 29	8 36 13·5	12 1 42·5
10	25 50	35 52	42
20	26 11·5	35 31·5	43
30	26 32·7	35 10	42·7
40	26 53·0	34 49·5	42·5
50	27 13·5	34 28·3	41·8
66 0	27 35·3	34 7·5	42·8
10	27 56·5	Lost.	....
20	Lost.	33 25·0	....
30	28 38	33 3·5	41·5
		Mean ..	12 1 42·28
Approximate time by watch at apparent noon ..			6 0 51·14

The above mode of proceeding is conveniently applicable when the regularity of performance of the assistant-watch can be depended on throughout the period embraced by the observations, for in this case the "equation of equal altitudes," and the "noon comparison with the standard," being applied to the above "approximate noon by watch," will give the time shown at noon by the standard.

If, however, the going of the assistant-watch cannot be implicitly relied on, it will be better to take the mean of the corresponding A.M. and P.M. observations separately; apply to each mean the comparison with the standard,* and thus obtain its times corresponding to the A.M. and P.M. observations: the half sum of these two times will be the approximate noon by the standard, to which the "equation of equal altitudes" is subsequently to be applied as before.

In noting the observations it is advisable, as a check at the time on their accuracy, to take the sum of the corresponding times, A.M. and P.M., in the margin of the memorandum-book in which the sights are recorded; these several sums should all

* See the remarks on the subject of the comparisons, in the section under that head, p. 58.

agree (each one representing twice the time of approximate noon by the watch). Their accordance, or otherwise, therefore, affords a criterion of the accuracy with which the observations have been made,—both of the degree of precision with which the observer has appreciated the contacts, and the assistant noted the times.* This practice will be found very useful, and should always be adopted by beginners; since, by affording an unerring test of their skill, it enables them gradually to acquire a proper but not undue confidence in the value of their observations.

It may be proper to remark here, that it seems advisable, as far as may be practicable, that an observer should always employ the *same* assistant to note the time for him: two persons accustomed to work together are less liable to fall into any error from the want of a mutual intelligence between them, and, after a time, it is reasonable to suppose that the effects of personal equation will approximate to a condition of constancy, both in quantity and direction; whereas, with a frequent change of persons, this element of error is in a state of perpetual change. Certainly the same assistant who commences a set of observations should be employed to complete them, and the times A.M. and P.M. should always be noted by the same party.

Many careful observers are in the habit of noting the contact of both limbs of the sun in taking "equal altitudes;" the advantage of this method being, that it has a tendency to neutralise the errors of observation arising from any personal habit in the observer, which may induce him, in estimating the moment of contact, to be either always a little too soon, or always a little too late: the chances being, that these errors tell in opposite directions, according as the observed limbs of the sun are approaching or receding.

The mode of observation is as follows:—The sextant is set to any convenient division, a little in advance, in point of time, of the true (double) altitude (that is, to a greater altitude in the forenoon and a less in the afternoon), so that the reflected

* It may also be useful to point out, that after the first two observations, the assistant who is taking the time becomes aware of the interval between the contacts (in the above example *about* 21^s), and is thus prepared to note the time of the succeeding contacts, looking out sharply as the moment approaches, without the necessity of any premonitory signal from the observer. The attention of the assistant thus concentrated more readily seizes the subdivision of the "second," and the attention of the principal is not distracted by superfluous speaking.

suns may be seen in the field of the telescope, separate from each other, and approaching. Then the stationary image seen directly in the quicksilver will always be the upper in the forenoon, and the lower in the afternoon; (when the inverting telescope is used, these appearances will be reversed). At the moment of contact of the limbs the time is noted, the reflected image of the sun then passes over the true image; and at the moment of separation the time is again noted: by this means two sights are obtained with only once setting the instrument.

The index is then moved through two degrees, which will allow ample time to prepare for the next set; after a little practice one degree and a half will be found sufficient. This matter being also in some degree regulated by the quickness of the sun's motion in altitude at the time; the observations are then repeated, and this is done in succession through five sets (or any convenient number). In the afternoon the observations are repeated in reverse order (the last set in the forenoon being the first set in the afternoon); the time by the assistant-watch being noted as before.

The observations, when completed, will stand recorded in the following manner:—

Altitudes. ° ′	Times, A.M. h m s	Times, P.M. h m s	Sums. h m s
70 0	5 19 17	10 31 20	15 50 37
	Lost.	....	....
72 0	5 25 31	10 25 5·5	36·5
	28 51·5	21 45	36·5
73 30	5 30 19·3	Lost.	....
	33 42·5	10 16 54·3	36·8
75 0	5 35 11	10 15 26	37
	38 39·5	11 57·5	37
76 30	5 40 9·7	10 10 26	35·7
	43 42	6 54·3	36·3
		Mean..	15 50 36·60
Approximate time by watch at apparent noon..			7 55 18·30

The corresponding times of the forenoon and afternoon observations are then to be added together, and their sums meaned;

the half of which mean result will be the approximate time by the watch at the moment of apparent noon; from which, by applying the "equation of time," and the noon comparison, the time by the standard at apparent noon is determined: or, as before pointed out, the comparisons of the assistant with the standard are to be applied separately to the A.M. and P.M. times of observation, and the time of the standard thus directly ascertained.

In high latitudes, when the sun's motion in altitude is very slow, it may not be convenient to wait for the sun to pass over its own diameter; in which case, five altitudes of each limb (or any convenient number), with their corresponding times, may be observed on each side of the meridian. The only way in which this method is inferior to the last is in having to shift the index twice as often to the same number of observations.

If cloudy weather should interfere to prevent observations being taken through a regular series of sets, as recommended in the foregoing methods, several independent sights should, if possible, be obtained, clamping the index at any convenient division, and carefully noting the times corresponding to each altitude: several observations may thus be noted of either limb. In the afternoon the observations are to be repeated in reverse order, noting the corresponding contacts of the limbs, care being taken that the same observations are made of each limb, and remembering that in the forenoon lower limbs will be found to separate, and upper limbs to close, the reverse taking place in the afternoon.*

As some of the corresponding times in the afternoon may be lost, it is advisable, in taking the mean of the sums of the A.M. and P.M. times, that an equal number of observations of each limb should be retained; superfluous ones should therefore be rejected, so as to equalize the numbers.

The observations, when completed, will stand recorded as follows:—

* In order to avoid any confusion between the two limbs, beginners will find it convenient to make their approximate contacts, and satisfy their minds on the matter in the first instance, without using any tube; the inverting tube being afterwards screwed in, when they are all ready to commence observing.

Altitudes.	Times, A.M.	Times, P.M.	Sums.
L. L.			
° ′ ″	h m s	h m s	h m s
97 45 40	8 50 44	2 13 39·5	23 4 23·5
98 17 30	51 54·5	12 29·5	24
98 53 10	53 13	11 11	24
99 28 20	54 29·5	Lost.	....
100 3 30	55 47	8 36·3	23·3
U. L.			
101 40 20	8 57 1	2 7 21·5	22·5
102 9 20	58 5	6 19·5	24·5
102 38 40	59 8·3	5 14	22·3
103 7 40	9 0 11·5	4 13·5	Rejected.
103 37 50	1 17	3 6·7	23·3
		Mean ..	23 4 23·42

Approximate time by watch at apparent noon .. 11 32 11·71

In the foregoing remarks we have supposed that, generally speaking, equal-altitude observations of the sun are taken by observing A.M. and P.M., and thus deducing the error of the watch at the moment of apparent noon: it may, however, be occasionally necessary or convenient to reverse the process, and take the observations, first in the afternoon, and then again on the forenoon of the following day. The mode of observation is exactly the same, except that they are taken in reverse order, the last of the afternoon being the first in the following forenoon. The reductions required are made in the same manner, and the final result is the time by the watch at the moment of apparent midnight.

In taking equal altitudes, in order that the observer may be prepared to get the return sights in the afternoon, without wasting unnecessary time beforehand in waiting for them, or running the risk of losing them altogether, if too late, it is advisable that he should compute *approximately* beforehand the time by watch when the first return sight is due.

The following convenient formula for this purpose has been proposed by Mr. Jeans:—

If t = time by watch when the forenoon observation was taken,
t' = time of corresponding sight in the afternoon,
λ = approximate error of watch or local mean time, + *fast* and — *slow*,

ε = approximate equation of time, + when *additive* to mean time, — when *subtractive;*

Then $$t' = (12^h - t) + 2(\lambda - \varepsilon).$$

On which formula it is only necessary to remark, that attention must be paid to the algebraic signs of λ and ε.*

Single Altitudes.—When want of time, the state of the weather, or other circumstances, do not permit "equal altitudes" to be taken, the method of "single or independent altitudes" must be resorted to.

The observations may be made in either of the three ways mentioned above, in treating of "equal altitudes;" that is, by setting the index to any given division, noting the contact, and then advancing the index successively through the following consecutive divisions, and repeating the operation; or by noting the contact of both limbs in succession, allowing the image of the sun to pass over itself; or by observing the contact of the

* The demonstration of this formula is as follows:—

Since t = time by watch, when the A.M. observation was taken,
$t - \lambda$ = mean time of A.M. observation,
and $t - \lambda + \varepsilon$ = apparent time of A.M. observation.

Hence $12^h - (t - \lambda + \varepsilon)$ = the interval to apparent noon, or, the apparent time of the P.M. observation,
$\therefore 12^h - (t - \lambda + \varepsilon) - \varepsilon$ = the mean time of P.M. observation,
and $(12^h - (t - \lambda + \varepsilon) - \varepsilon) + \lambda$ = time by watch of the P.M. observation, or t'.

Hence reducing
$$t' = (12^h - t) + 2(\lambda - \varepsilon).$$

Ex. Suppose, in the last set of observations quoted above, that the approximate error of the watch was 18^m slow on mean time, and the equation of time 10^m additive to mean time,

Then,

$$\left.\begin{aligned} t &= 9^h\ 1^m \\ \lambda &= -18 \\ \varepsilon &= +10 \end{aligned}\right\} \quad \begin{aligned} 12 - t &= 2^h\ 59^m \\ 2(\lambda - \varepsilon) &= -56 \\ t' &= 2\ \ 3 \end{aligned}$$

Hence, if the observer had been at his post, at say $1^h\ 55^m$ P.M. by his watch, he would have been in time enough.

The formula is equally applicable to the case of P.M. and A.M. observations, if t represents the first, or P.M. observation, and t' the last, or A.M. one.

Ex. (Raper, "Practice of Navigation," p. 274):—

$$\left.\begin{aligned} t &= 5^h\ 21^m \\ \lambda &= +1\ 55 \\ \varepsilon &= -\ \ 14 \end{aligned}\right\} \quad \begin{aligned} 12 - t &= 6^h\ 39^m \\ 2(\lambda - \varepsilon) &= +4\ 18 \\ t' &= 10\ 57 \end{aligned}$$

which agrees with the recorded observations. (See Jeans' "Navigation," &c. p. 189.)

limbs at any convenient division whatever, an *equal* number of observations of each limb being made as rapidly as circumstances permit.

In the two latter cases the mean of the altitudes observed will be the mean altitude of the sun's centre, corresponding to the mean of the respective times, all reference to the sun's semi-diameter being eliminated; and in the former case, the mean of the observed altitudes will be the mean altitude of the limb observed, corresponding to the mean of the respective times as before.

Either of these three methods of observing seems preferable to the practice of making the contacts by moving the tangent-screw up to the instant of observation, because there is always a tendency to error from the spring or elasticity of the index-bar;* and, moreover, as Raper observes, moving the tangent-screw diverts a portion of the attention which should be devoted to the contacts alone.

When only one limb is observed, it has been recommended that the lower limb should be observed when the observations are taken in the forenoon, and the upper when they are taken in the afternoon; because the images of the sun are then receding, and the contact of the limbs, at the moment of separation, can be more correctly observed than when they are approximating.

Since, in nice observations for time, the corrections to the observed altitudes for refraction should be accurately applied, the state of the barometer and thermometer at the time of observation should be carefully noted. Even when "equal altitudes" are taken, this precaution should be adopted, as from failure in obtaining the return-sights in the afternoon from cloudy weather, or other causes, the possibility of having subsequently to reduce the forenoon observations as single altitudes should always be contemplated.

* The error from this cause is often different for the *onward* and for the *backward* motion of the index. As a remedy for this, it has been proposed that all observations should be taken with the same motion of the index-bar. "The *onward* motion being adopted as the most natural, the tangent-screw is always employed to close the object and the reflected image, and is thus always turned in the same direction." ("Practice of Navigation," p. 164.) When, however, circumstances permit, as in the observations under present discussion, it seems preferable to avoid using the tangent-screw altogether in making the contacts.

Circumstances may occasionally render it more convenient to obtain the time by observations of stars at night, rather than by altitudes of the sun by day. In this case, equal-altitude observations are rarely practised, on account of the inconvenience of having to wait so long at unseasonable hours for the duplicate observations *west* of the meridian. When, however, they are resorted to, the observations determine the time with much precision, since the declination of the body observed being invariable, there is no equation of equal altitudes to be applied. Independent altitudes of stars of nearly equal declinations, and nearly equidistant from the meridian, both *east* and *west*, are more usually had recourse to; the best mode of taking them being by clamping the index a little in advance of the true (double) altitude, when the body is east of the meridian, and a little in defect of it when west, waiting in both cases till the motion of the star in altitude produces a contact; the time is then noted, and the instrument read off; the operation being repeated any convenient number of times to neutralise the small errors of observation. The best way of appreciating the moment of contact is by slowly moving the sextant round the axis of vision, making a sweep with the reflected image of the star, until it is observed exactly to coincide with the real image as it passes over it.

Comparisons with the Standard.—Since, as we have before remarked, the chronometers are on no account ever to be removed from their box in the "chronometer-room," the time corresponding to all observations made on shore is always to be taken in the first instance by the assistant-watch, and the corresponding indication of the "standard" at the same moment subsequently deduced by means of the comparisons. As the standard chronometer and the assistant-watch will rarely have the same rate, the comparisons before and after may exhibit an appreciable difference; and in this case the required comparison at the moment of observation must be obtained by interpolation, on the supposition of an equable variation in the rate of change.* Even supposing the whole or a part of the variation to have been owing to real irregularity in the going of the assistant, to which even good watches are sometimes prone, when subjected to motion and careless usage, it is best to adopt this idea, as it is, of course,

* See *ante*, p. 37.

impossible to say at what precise moment, whether before or after the observation, that the irregularity, or *jump*,* took place. This plan, moreover, at any rate, distributes the effect of the irregularity over a larger period, and, in the absence of any direct knowledge on the point, is free from the objection of an arbitrary evasion of the difficulty.

When equal-altitude observations are taken, the "assistant" should, therefore, be compared with the standard, both before and after the observations A.M. and P.M.; the corresponding times shown by the standard at the moments of observation are thus obtained: the mean of these times, when corrected by the "equation of equal altitudes," gives the time shown by the standard at the moment of apparent noon; and from this time and the "equation of time" the error of the standard, on mean time at the place at apparent noon, is at once obtained.

In order to obtain the errors of the other chronometers, it is customary to get a comparison with the standard all round, about the time of apparent noon; and these several comparisons being then applied to the error of the standard, as pointed out before (see *ante*, p. 33), will give the respective errors of all the other chronometers at that instant.

When the time is obtained by single altitudes, the indication of the standard at the moment of observation is in like manner obtained from its comparisons with the assistant. In many ordinary cases, including the determination of the longitude by chronometer at sea, when no very rigid accuracy is required or sought for, a single comparison with the standard, either before or after the observations as may be convenient, will suffice to give its time with sufficient precision; and in the case of forenoon observations, the usual daily 8 A.M. comparison at the time of winding will, no doubt, be all that is necessary for all practical purposes; and a similar plan will answer for the several other chronometers when required.† If, however, the

* There is reason to suppose that the "jumps" which sometimes take place in watches and chronometers, and which are often mysterious and perplexing, proceed from defects in the jewels on which the pivots of the movable parts work. ("Rech. Chron." p. 281.)

† When a ship is furnished with several chronometers, as it would be troublesome throughout a long voyage to deduce the longitude from each of them daily, it will be quite sufficient to keep the daily reckoning by the "standard" only. The indications of the other chronometers, from their known errors and rates, may then be obtained, for the purpose of comparison with the standard, once a-week, and their

time by the several chronometers is accurately wanted for some special purpose, this approximate mode of proceeding will not be sufficiently exact, and it will be advisable to get a round of comparisons of each of the chronometers with the standard, both before and after the observations, so as to obtain by interpolation the exact comparisons at the required moment. In the case of forenoon observations, the usual morning comparison at the time of winding may always be employed as one of them.

When it is proposed to determine the time by "equal altitudes," it may frequently happen that the observer fails to obtain duplicate sights in the afternoon, and hence the forenoon sights, and also, perhaps, any independent ones that may be obtained in the afternoon, have to be subsequently reduced as single altitudes; in this case, the required comparisons of the several chronometers with the standard, at the moment of observation, may be obtained for the forenoon observations, by interpolating between the comparisons with the standard, at the time of winding and noon, and those for the afternoon sights from the noon comparison, and a special one made after the completion of the afternoon operations on return on board, say at about 4 P.M.

The system of double comparisons and the consequent interpolations here recommended, may appear at first sight very troublesome, and almost unnecessary; but where in delicate measurements rigid precision is sought for, we are persuaded that it will, in the long run, be deemed much more satisfactory than being obliged boldly to assume, from time to time, that the comparisons have not altered during long intervals; or than being compelled to interpolate for intermediate values between two consecutive *daily* comparisons, which will otherwise be the only alternative.

Position of the Place of Observation.—In selecting an appropriate place for observations for time, much must often depend on circumstances at the moment, over which, perhaps, the choice of the observer has little or no control. If the place is a well-

individual differences from the indications of the standard at that time may afterwards be applied as corrections to its results during the ensuing week, in case their values of the longitude are required to be known. In a similar manner, the weekly value of the reduction of the standard to the "general mean" may also be obtained.

known station, often visited by other navigators, and recognised in published documents as a hydrographic position, it seems advisable, when practicable, that new visitors should make their observations at the same spot, which has already been selected by previous observers. There is no advantage whatever in the unnecessary multiplication of sites of observation; on the contrary, the practice tends to great confusion, and interposes difficulties in the comparison and incorporation of the results obtained by different navigators. If the spot of observation used by former observers be not known, or for some sufficient reasons not then conveniently accessible, then each new visitor may be at liberty to choose a place at his own convenience; and in this case it is of importance that he should be careful to describe its situation accurately, and especially to give its true bearing and distance from, or the reductions in, latitude and longitude, which may be necessary to connect it with some salient point, whether natural or artificial, used in the description of the place by former voyagers, or adopted in published plans or charts, and therefore easily recognised: such as a Peak, Point, Rock, Flagstaff, Lighthouse, or Church, &c. &c.

If the place visited be a new station, it is desirable that the place of observation selected should be generally convenient, and easy of access, and that it should be accurately described by a reference to such natural or artificial objects as exist in the vicinity, so that subsequent visitors may have no difficulty in recognising the place again from its published description.

The latitude of the place of observation should be known accurately, because it is a direct element in the computation of the observations for time. The longitude of the place of observation is required approximately, as an element in the reduction of the data required from the "Nautical Almanac;" such as the "sun's declination," the "equation of time," &c.

If the latitude of the place be not known, it must be determined. The best and most practical way of obtaining it speedily, and with facility, is by the meridian altitudes of stars observed in the artificial horizon. If an equal number of stars be observed north and south of the zenith, this mode is susceptible of great accuracy, as the errors of observation, both those which arise from defects in the instrument and those which are per

sonal to the observer, have a tendency to neutralise one another, and a skilful observer may thus in one night often obtain his latitude with a great degree of precision.

The approximate longitude, if not otherwise known, may be obtained sufficiently exact for the purposes required, by bringing it forward by the chronometer from the last station left.

If any published plan or chart of the place exist, issued by competent authority, and of established reputation, no doubt the latitude and longitude of some prominent station in the plan will be given in it, and the reduction in latitude and longitude of the spot of observation to this point may easily be obtained from the scale and by measurement.*

Reduction of the Observations.—Ample rules for the computation of the time, both by the method of "equal altitudes,"† and that of "single altitudes," are to be found in the various treatises on navigation in use among seamen; and the persons into whose hands these pages may fall, and to whom they are likely to prove useful, will doubtlessly be perfectly conversant with the usual methods of computation employed. Having, therefore, nothing new to offer in a track so well beaten, we shall, in referring our readers to the rules laid down by the

* The required reductions may be effected approximately by the "traverse table" as follows:—

With the true bearing of the spot of observation from the meridian of the known position as a *course*, and the distance (expressed in miles) as a *dist.*, enter the table. The corresponding *diff. lat.* will be the difference of latitude required, and the *dep.* the departure required: then, with the latitude as a course, and the *dep.* in the *diff. lat.* column, the corresponding *dist.* will be the difference of longitude in minutes of arc. If required in seconds of time, multiply it by four. (*Note.*—The number of feet in a "nautical mile" may be assumed as a mean value at 6075.)

If wanted more accurately, the reduction may be obtained by the following formulæ:—

$$\text{Diff. lat. in minutes of arc} = [6{\cdot}216454]\ \alpha \cos.\ \theta.$$
$$\text{Diff. long. in minutes of arc} = [6{\cdot}216454]\ \alpha \sin.\ \theta\ .\ \text{sec. lat.}$$
$$\text{Diff. long. in seconds of time} = [6{\cdot}818514]\ \alpha \sin.\ \theta\ .\ \text{sec. lat.}$$

Where α = true distance in feet, and θ = true bearing of the place of observation, from the known station.

The quantities inclosed in brackets are the logarithms of the numerical constants which enter into the formulæ; in using the formulæ the tens are to be rejected from the indices of the sums of the logarithms.

† In many of the treatises on navigation used by seamen, the rules for the computation of the "equation of equal altitudes" involve the use of proportional logarithms. Such an arrangement is not suited to cases where all possible accuracy is

established writers on this subject, simply confine ourselves to a few practical remarks and observations tending to enforce careful habits of computation, and the accurate realisation of results.

In reducing data from the "Nautical Almanac"*—the "Greenwich date" having previously been obtained to the nearest minute—the quantities should be carefully computed, as closely as the construction of the tables admits; the sun's declination, and its daily change, to the nearest second; and the equation of time to two places of decimals of a second. If star observations have been used to determine the time, the "right ascension of the mean sun" should in like manner be accurately reduced, and the "stars' right ascension" taken for the given day from the table of their apparent places in the ephemeris, and not from the table of their mean places, which only gives their approximate values.

The latitude, true altitude, and declination (or their complements, as the case may be), should be expressed to the nearest second; the interval of elapsed time (required as the argument for the tables of A and B, for the reduction of "equal altitudes") will be sufficiently exact to the nearest minute. The logarithms of the various quantities employed should be taken out accu-

required, and it seems then better to make the calculation by the aid of tables of common logarithms: rules adapted to their use are given in Riddle's and Norie's Navigation, and also in Raper's (in a note, p. 273).

Sometimes the rules for assigning their proper signs to the two parts of the equation are so expressed as apparently to involve an unnecessary number of distinction of cases. It will appear on examination that the following rule holds good:—

The sign of the first part of the equation is *positive* from the summer to the winter solstice, and *negative* from the winter to the summer solstice: (according to the hemisphere in which the observer is: that is, considering the summer solstice to be in December, and the winter solstice to be in June, when the observer is in south latitude). That of the second part *positive* between the equinoxes and the solstices, and *negative* between the solstices and the equinoxes: (but if the elapsed time is greater than twelve hours, the sign of the second part must be reversed); the advantage of which rule is, that it is expressed as a simple function of the season of the year, and is apparently independent of mathematical conditions.

* It is desirable that the quantities required should always be taken from the "Nautical Almanac," and not from the rough tables given for the purpose of superseding its use, which are to be found in many of the treatises on navigation: such tables, although, perhaps, useful enough for rude purposes in ordinary navigation at sea, are wholly unsuited to the delicate determination of time from observations on shore.

rately, and proportional parts obtained for odd seconds, &c., when necessary, by interpolation.*

All results in time, the equation of equal altitudes, hour angles, errors of chronometers on time, comparisons with one another, and rates, should be carried to two places of decimals; not that it is for one moment either supposed or pretended that in practice such a degree of minute accuracy is either really attainable or necessary, even if it were possible, but theoretic considerations seem strongly in favour of such refinements as applied to computation. In the first place, there can be no doubt that precision of reduction, and closeness of work, tend to encourage in the computer accurate habits of calculation; secondly, it should be remembered, that the results sought for are often minute quantities, and it is vain to expect that they should be obtained satisfactorily if the means employed are only rude and approximate; and lastly, the practice may be fairly defended by a consideration which may be aptly alluded to by parodying a well-known aphorism; and we may remind our readers, that if we look after the hundredths and tenths, the seconds and minutes will take care of themselves. In thus recommending closeness of reduction and calculation, one caution, however, is necessary,—the computer should ever bear in mind that no degree of accurate computation can ever compensate for any inherent defects, or erroneous assumptions, which may exist in the data employed. The influence of such errors will make itself felt, whether the subsequent computations are performed closely or rudely; but it is not unreasonable to expect that the practice of precise calculation may reciprocally engender in the observer the habit of obtaining the elements of his operations as accurately and as free from errors as circumstances permit.

Preconcerted Signals.—Sometimes, when it is not convenient to obtain the errors of the chronometers on time by means of

* Many logarithmic tables contain either proportional parts or columns of differences, to enable the student to compute to the nearest second. If such tables are not at hand, or their use not approved of, the computer may easily make the necessary reduction himself, by taking the difference between the consecutive logarithms as they stand in the tables, and obtain the proportional part required for the seconds in excess by a short calculation by practice, or the rule of three.

independent astronomical observations, advantage may be taken of the arrangements existing for that purpose at various ports, for obtaining them by preconcerted signals; or special arrangements for that object may be devised for the occasion.

Under the general term, "preconcerted signals," we mean to include the various arrangements which exist at several ports and places for notifying at precise moments the mean time at the place, by the fall of time-balls, the display of signal-flags, the flashes of guns, &c. &c.; and also those special measures which may occasionally be devised between different ships for the intercomparison of their chronometers.

The arrangements for the former class of signals will, of course, vary very much at different places; and it is of importance, in order that the seaman may be able to avail himself fully of the advantages of this plan, that he should make himself fully conversant by inquiry with the nature of the local regulations on the matter. Sometimes the final signal is preceded a short time before by some distinctive announcement, to call the attention of observers; sometimes a premonitory signal, at a given interval, precedes the final one; sometimes the signals, of whatever nature they be, are repeated two or three times, for the convenience of observers.

When the fall of time-balls is regulated by galvanic apparatus in connexion with the Mean Time Clock of an established observatory,* the fall of the ball at the precise pre-arranged and expected moment may be implicitly depended on. When this, however, is not the case, when the ball is liberated by hand, or by some ordinary mechanical appliance, or when the time is indicated by the flash of a gun fired at (*about*) some pre-established instant,† the exact moment of the occurrence of the phenomenon is usually subsequently published for general information. All these matters, depending on local arrangements, should be duly inquired into by those desirous of profiting by them.

In noting these phenomena on board the ship, it is desirable that, when circumstances permit, the time should be at once taken by the "standard," and that the intervention of an intermediate

* As at Greenwich and Edinburgh, for instance.

† As at Madras, where the evening gun is fired from the Fort at 8 P.M., and the true mean time of the flash, by the Observatory clock, subsequently published in the Government Gazette.

watch should, if possible, be dispensed with. Notwithstanding the apparently inconvenient position of the chronometers, low down in the interior of the ship, while the signals have to be observed from the deck, there will generally be no great difficulty in attaining this end, if the arrangements for the observation are properly organised, and *discipline* interpose to secure the needful *silence* during the brief period occupied by the operation. The signals may be transmitted from the observer on deck to the one stationed at the chronometer, either *sharply* by word of mouth, or by a *smart* blow on the deck by a stick or by the foot.

Sometimes it may be necessary or convenient to compare the chronometers with those of other ships. In this case, naval ingenuity will have no difficulty in arranging means by which flags giving the required signals should be *smartly dipped;* or, better still, in devising for the occasion an impromptu time-ball. Again, by a proper system of *leading blocks,* it is easy to arrange, that the observer at the chronometer on board the ship giving the signals, should himself give the instant of observation, by letting go from his hand the rope suspending the ball; thus making the signal more instantaneous, and dispensing with the intervention of a second party.

Sometimes, when observations for time are made on shore, the place of observation may not be conveniently accessible from the ship; or from the state of the weather, or other causes, communication with the ship may be difficult or impracticable: in this case, the comparisons of the assistant-watch with the standard must be made by signals in the manner alluded to above.

In all cases of comparisons between ship and ship, or from the shore to the ship, it is important that the system intended to be acted on be thoroughly understood by all parties concerned; and when the operations are completed, the parties should compare notes as soon as possible, so that discrepancies should be at once detected, and suitable measures adopted for rectifying them when necessary.

In all cases of the intercomparison of the chronometers of different ships, the comparisons are supposed to be made in the first instance between their "standard" chronometers. A round of comparisons of each of the other chronometers with their respective "standards" should then be taken immediately afterwards, if their indications are also required.

CHAPTER IV.

ON RATING CHRONOMETERS, AND ON THE DETERMINATION OF THEIR ERRORS ON LOCAL MEAN TIME—SIMPLE METHOD; BY DUPLICATE OBSERVATIONS OF A SIMILAR CHARACTER, TAKEN AT A CONVENIENT INTERVAL.

THE chronometers having been received on board, and carefully stowed, in the manner previously recommended, in the places allotted for their reception, it next becomes of importance to ascertain their rates, and also their errors on local mean time.

It is frequently customary to adopt, at the commencement of a voyage, the errors and rates received with the chronometers from their makers, or from their place of deposit, or rating on shore; it being assumed that these data, which, no doubt, have been determined with a proper regard to accuracy, may be sufficiently depended on for all practical purposes, until more ample leisure and a favourable opportunity afford ready facilities for the redetermination of these elements. For the rough and approximate requirements of the ordinary processes of navigation there may, perhaps, be no great harm in thus confidently assuming the permanency of the data thus received; but if it is proposed to employ the chronometers for higher uses than their application to the daily navigation of the ship, and if an ambition exists to make them subservient to the more exacting requirements of science, and to obtain from them satisfactory contributions towards the improvement of hydrography, it will not be proper to rely on the stability of their rates previously obtained on shore; while at the same time the greater importance of the results sought for, demands, and should receive, a more careful attention to the conditions necessary to ensure accuracy.

It is found in practice, that the rates of chronometers, after they have been received on board ship, rarely agree with those they previously had when on shore; the causes of this variation

are probably to be sought for in the disturbing effects of accidental circular motion, or slight shocks during the process of removal, in the alteration of temperature to which they have been subjected, and in the possible effects of magnetic influences to which they may become liable in their new position: be this as it may, it seems always advisable, if possible, when accuracy is aimed at, to rate the chronometers anew before starting on the voyage, after they have been some days on board, and after they have become, if we may so express it, *naturalised* in their new position, and have settled down to a stability of rate under the new conditions to which they are submitted. It is with this object in view that, in a preceding chapter, we have recommended their being received on board, during the equipment of the ship, at a much earlier period than is generally customary.

By the "rate of a chronometer," we mean the difference of its error on local mean time from day to day. It is called *gaining*, when it goes too *fast*, and *losing*, when it goes too *slow*.

The usual method of obtaining the rate consists in determining by duplicate observations of a *similar* character,* taken at a convenient interval, the error of the chronometer on mean solar time at the place. If at the period of the second observation it be found that the error of the chronometer remains unchanged, it shows that the chronometer has no rate, and is, therefore, keeping true mean solar time; if, on the other hand, it appear that the error on local mean time is no longer the same, it shows that the chronometer has a sensible rate, and

* By "observations of a similar character," we mean observations similar, as far as may be, in their mathematical conditions, and therefore liable to be similarly affected by the possible errors in the data on which they depend.

For instance, in the comparison of observations for the deduction of errors and rates, it is of importance that equal altitudes should be used in combination with equal altitudes, because any possible errors in the assumed latitude of the place of observation would be neutralised in its effects, in so far as the rate is concerned, while it would equally affect the absolute errors on time; and, at the same time, it is not unreasonable to suppose that the errors arising from any personal defect of the observer would equally vitiate the two observations.

In a similar manner, it is fair to suppose, that since they are taken at nearly the same time, and under similar conditions, the errors of refraction and observation would probably equally affect single-altitude observations taken A.M. at any two periods conveniently consecutive; and that they might, therefore, with propriety, be used together for the determination of errors and rates. Similar reasoning applies to the combination of two P.M. observations taken consecutively.

If, in default of equal altitudes, the mean of the times obtained from A.M. and P.M. sights, at nearly equal intervals from noon, be used, they should, in like manner,

the difference* of the two errors divided by the interval elapsed between the observations expressed in days will give the "mean daily rate" of the chronometer during that interval: which rate, moreover, it is important to remark, in strictness belongs to the epoch corresponding to the middle of the interval between the observations.

If the chronometer were slow on local mean time at the first observation, and less slow at the second, or fast and more fast, it shows that the rate of the chronometer is +, or *gaining*. If, on the other hand, it were fast at the first observation, and less fast at the second, or slow and more slow, then the rate of the chronometer is —, or *losing*.

In indicating the condition of the error of a chronometer, it will be convenient, as tending to facilitate subsequent algebraic discussion, to consider errors *fast* on local mean time as *positive*, or +, and errors *slow*, as *negative*, or —. Of course, mathematically considered, it is immaterial and altogether arbitrary, whether we choose to consider a chronometer *fast* on local mean time by a given quantity, or its complement to 12 hours *slow*, or vice versâ, *slow*, or its complement *fast*: thus, if at noon at a given place a chronometer show $9^h\ 54^m\ 11^s$, it may either be said to be that quantity *fast*, or its complement to 12 hours, viz. $2^h\ 5^m\ 49^s$ *slow*. In practice we apprehend it will be found more convenient to consider all the chronometers *slow* on local mean time when voyaging in *east* longitude, and fast when in *west*. When the measurement of meridian distances is pro-

be combined with similar data obtained on a subsequent occasion: but inasmuch as the small errors of refraction and observation are probably neutralised in the final results, there does not seem to be any particular objection to comparing the mean of results by A.M. and P.M. observations with those obtained from equal altitudes on a previous or subsequent occasion, provided the value of the latitude used in the calculations can be accurately depended on.

In like manner transit observations should be compared with transit observations; and generally, of whatever nature the observations for the determination of the time may be, they should be referred to corresponding observations of a similar character.

It should be borne in mind, that the general tendency of this mode of proceeding will be to neutralise the effects of the errors of observation on the *rate*, and to make it a constant on the *error* on local mean time, since the former is obtained by comparing the difference between given observations, and the latter from the observations themselves, or by taking their mean.

* This difference is their *algebraic* difference; therefore, when the error is found to have changed from fast to slow, or from slow to fast, the rate is the sum of the errors divided by the number of days elapsed.

posed to be undertaken, some uniform arrangement of this nature will be found very convenient; and as, moreover, it is reasonable to suppose, that before the commencement of the voyage the chronometers were originally adjusted approximately to Greenwich mean time (or that of the first meridian employed), the plan here recommended will maintain that analogy, whatever apparent discrepancies may ultimately exhibit themselves through the gradual divergence due to the diversities of their rates,—whichever way the matter may be viewed, it is of importance that *all* the chronometers be treated alike, and that the plan originally adopted be steadily adhered to throughout.

For the determination of the error on local mean time, it has formerly generally been the practice in the measurement of meridian distances, or on taking a departure from a port, to adopt as a starting-point the error shown by the last observation used in the determination of the rate; but inasmuch as, in the deduction of the latter, we necessarily place equal confidence in both the observations, there seems to be no good reason why we should not do the same in assuming the error, and adopt as our *working* error the mean of the two errors on which we have already agreed to make the rate depend.

The *mean* error thus introduced may probably be assumed to be more accurate than either of the single elements on which it depends; while this mode of proceeding, moreover, will have the advantage of referring both the error and the rate to the same epoch: a circumstance which, when we come by and by, in the discussion of the formulæ for meridian distances, to treat of the corrections to be applied for the changes of rate which may have taken place in the transit from port to port, will be found to introduce considerable simplicity into the equations employed.*

The immediate result of any observation, at any place, for the state of the chronometer at any given moment, will be its error on local mean time. This error, depending on observa-

* On this subject M. du Conulier observes, "Conformably to a remark already made by M. Barral ('Annales Maritimes,' Jan. 1829). It is the mean day of the observations and not the last, that we ought to take, for that on which the daily rate of the watch has been determined; and it is to this mean day, that one ought to refer its absolute error." ("Rech. Chron." p. 108.) We may add that this practice also had the approval of the late Mr. Raper.

tions having reference to astronomical or other data, will be true within the limits of the correctness of the elements employed; and in any particular case, by multiplying the observations, or by adopting specific practical precautions suitable to the occasion, a great part of the probable errors may generally be almost entirely eliminated, and the residual error reduced within very small limits: and it is important to remark, that the error on local mean time thus obtained is wholly independent of any previous arbitrary assumption of the actual longitude of the place of observation.*

In the measurement of meridian distances, which is simply a careful comparison of the difference of time under two meridians, it is of importance that the question should not be complicated by any reference to the relation which either place bears to Greenwich or any other "prime meridian."

In the development of the science of hydrography, the questions of the relative longitudes of outlying stations must be settled by the navigator, that of absolute longitude belongs exclusively to the hydrographer. The former carefully measures differences of longitude, and reports their results; the latter compares results, collates authorities, settles from the evidence of astronomical data the positions of the "secondary meridians," and unites the whole in one harmonious combination. It is, therefore, much to be desired, that navigators would solely confine their attention to the accurate measurement of differences of meridians by comparing the errors of their chronometers on local mean time at the two places, without any reference whatever to what the actual longitudes of the places of observation may be.

It is true that by carefully carrying on a chain of meridian distances from the commencement of a voyage, a consistent series of the values of the longitude of each place visited may be obtained. The values of all the longitudes thus deduced are all equally affected by any primary error which may exist in the assumed longitude of the place originally started from; while each particular longitude is, in addition, affected by the algebraic sum of the small errors existing in the individual links

* Except in so far as the reduction of the quantities required from the astronomical ephemeris is concerned, on which the value of the assumed longitude of the place of observation exercises a certain influence.

in the chain of which it is composed. It is reasonable to suppose that the algebraic sum of such small errors may, in well-executed measurements, frequently be very small, and that the minute errors of the several parts may often nearly eliminate one another from the contrariety of their signs; but there can be no doubt that the value of such a series, as a contribution to accurate hydrography, is greatly enhanced by viewing its constituent parts separately, link by link, and comparing their values with those of other authorities, rather than when, taking the chain as a whole, it is attempted during the successive stages of the voyage to settle the values of the absolute longitudes of the places visited. Pursuing the analogy suggested by likening the series to a chain, it should be remembered that as one faulty link renders a chain valueless, while the several other links taken separately may be perfectly good; so also one unsatisfactory measurement vitiates the value of a series of meridian distances, in settling chronometrically the actual longitude of any given station, while, at the same time, the other several parts of which it is composed may be unexceptionable in character, and of great and acknowledged value.*

The mode of proceeding, moreover, here recommended, has another great and decided advantage; each measurement of the meridian distance between two places visited during the

* The extensive series of meridian distances measured by Capt. Fitzroy, in H.M.S. Beagle (1831–6), may be quoted in illustration of the above remarks. The Beagle circumnavigated the globe, sailing from east to west, during a voyage occupying nearly five years. She was furnished at starting with twenty chronometers, which number, from various causes, was reduced to eleven at the termination of the voyage. The whole sum of the meridian distances exceeded 24^h by about 33^s. It is clear, therefore, that errors exist somewhere; the separate measurements, comprising the links of the chain, when examined one by one, appear unexceptionable in point of character, and remarkably accordant with those of other authorities. And what makes the ultimate error more perplexing is, that it is in the highest degree improbable that the ordinary errors of observation and the discrepancies to which chronometric measurements are liable, should all lie in the same direction, and that no compensation of error should take place from the contrariety of their algebraic signs.

Capt. Fitzroy suggested, that some unknown effects of magnetism, on account of the ship's head being a long time in a given direction, may possibly have affected the performance of the chronometers. This point is well worthy of future investigation, and its consideration is suggestive of the interesting results to science that might arise if a ship were sent on a scientific voyage charged with measuring a series of meridian distances round the world in both directions, from east to west; and, again, from west to east. Rio Janeiro, or Bahia, would be a convenient starting-point; and with proper arrangements, the double voyage might easily be performed in the course of three years. Other objects of scientific interest might be united with it.

voyage, (made in the manner to be hereafter explained), is complete within itself, and has no necessary connexion with those that may precede or follow it. This consideration permits the operations connected with the chronometers to be carried on with greater freedom, and relieves the persons engaged in them from any feeling of irksomeness or restraint. When circumstances are unfavourable, and pressure of business, adverse weather, or other cause, prevent the necessary observations, the work may be dropped for the time; on the other hand, when fortune is again propitious, and ample leisure, fine weather, and other favouring contingencies invite the undertaking, advantage may again be taken of the opportunity offered, and the measurements resumed.

For the ordinary purposes of navigation, a reference to the absolute longitude of the place of observation from Greenwich, or other *prime* meridian for which the charts employed are constructed, is, of course, indispensable; and the error of the chronometer on local mean time having been obtained as an immediate result from the observations in the first instance, its error on Greenwich mean time is subsequently deduced from it, by applying thereto the longitude in time of the place of observation;* but it is important that the navigator should bear in mind that all longitudes at any time obtained, and all

We cannot refrain, in closing these observations, from remarking that Capt. (now Admiral) Fitzroy is deserving of the highest praise for the thorough honesty with which he reported the results of his chronometric measurements, although apparently reflecting on his own success. How easily, by a little "cooking" and "trimming," judiciously paring off a second or two here, and a second or two there, and thus altering the estimated means, the results might have been "squared in," so that the final sum should have been exactly 24 hours, neither more nor less; or, perhaps, two or three seconds over or under, for conscience sake, or to give the appearance of extreme honesty! A sciolist would have yielded to this temptation: happily, for the credit of science, this officer was superior to such influences. (See "Voyages of Adventure and Beagle," Appendix, vol. ii. p. 331.)

* The general expression for the chronometric difference of longitude, or the meridian distance between any two stations, is

$$M = \lambda' - \lambda$$

(*See remarks on this subject in Chap. VII.*), in which expression M represents the meridian distance, while λ' and λ represent the errors of the chronometer at the same moment, on local mean time, at the two places, the former referring to the place arrived at, and the latter to the place sailed from, and being considered *positive* when *fast*, and *negative* when *slow*; hence, M will be *positive* when measured towards the *west*, and *negative* when towards the *east*.

If the place to which λ refers be Greenwich, then M represents the longitude of

positions in longitudes which it may be his duty to report during the conduct of his voyage, depend on, and will be vitiated by, any error in the assumed longitude of the place where his last error was obtained; hence, in making any report of this nature, in which he may mention his chronometric value of the absolute longitude of any place, the assumed longitude of his starting-point should be specified; or better still, for hydrographic uses (as we have before remarked), if he should content himself with giving the *difference* of longitude from the place where his error was last obtained.

The observations usually employed by navigators for the determination of errors and rates may be comprised under the following heads:—

I. Transit observations.
II. Equal altitudes.
III. Single altitudes, A.M. or P.M.
IV. Single altitude A.M. and P.M. observations combined.
V. By comparison with a time-ball, flash of a signal gun, or rocket fired at a given moment; with an observatory clock or with another ship, by preconcerted signals.

Of whatever nature the observations may have been from which the time at the place has been determined, the subsequent deduction of the errors and rates from them involves the same processes, and is similar for any number of chronometers employed.

We shall proceed to give some numerical examples:—

Example 1. By equal-altitude observations at Trincomalee Dockyard, on June 2d, 1851. The errors of chronometers Z, R, and Y, on local mean time, were as follows:—

the place to which λ' refers, and λ' and M being known, λ the error of the chronometer on Greenwich mean time, becomes known by transposition, since

$$\lambda = \lambda' - M.$$

In applying which formula to actual practice, attention must be paid to the algebraic signs, *west* longitudes and errors *fast* being considered *positive*, and *east* longitudes and errors *slow* being taken as *negative*.

It will be observed, moreover, that λ and λ' represent respectively, for the several chronometers employed, the quantities elsewhere denoted by Z − L, A − L, B − L, &c.; or by L − Z, L − A, L − B, &c. (See *ante*, p. 33.)

	h	m	s
Z —	5	29	54·23
R —	0	9	33·23
Y —	5	37	43·23

On June 9th, by similar observations, the errors were,—

	h	m	s
Z —	5	29	35·38
R —	0	14	28·68
Y —	5	38	23·58

Subtracting the respective errors of the chronometers on June 2d from their corresponding errors on June 9th, the differences are as follows :—

	s
Z +	18·85
R —	295·45
Y —	40·35

The interval between the epochs of the observations is 7 days ; hence the above differences, divided by 7, will give the *mean* daily rate during the interval, while the mean of the respective errors gives the mean error corresponding to the mean epoch for which the rate has been determined.

Hence, in this case, we have at the mean epoch, June $5^{d}\cdot5$:—

	Errors.			Rates.
	h	m	s	s
Chron. Z —	5	29	44·80	+ 2·69
R —	0	12	0·95	— 42·21
Y —	5	38	3·40	— 5·76

If, for the convenience of the ordinary processes of navigation, the errors of the chronometers on Greenwich mean time were required, they would be obtained from the preceding errors on local mean time, by applying the longitude in time to them, by the formula

$$\lambda = \lambda' - \mathrm{M}.$$

According to the most approved authorities, M, the longitude of the Dockyard flagstaff at Trincomalee may be assumed as $-5^{h}\ 24^{m}\ 53^{s}\cdot8$. Applying this to the above errors of the chronometers on local mean time, we shall have for their corresponding errors on Greenwich mean time as follows :—

		h	m	s
Chron. Z	—	0	4	51·00
R	—	6	47	7·15*
Y	—	0	13	9·60

Example 2. By single-altitude observations, taken at 9 A.M. on Feb. 6th, 1844, the errors of chronometers A, C, and I, on mean time at Garden Island, Sydney, New South Wales, were as follows:—

		h	m	s
A	—	10	1	28·10
C	—	8	57	21·55
I	—	11	9	10·75

Similar observations on Feb. 17th gave the respective errors as follows:—

		h	m	s
A	—	10	1	13·87
C	—	8	55	32·27
I	—	11	10	7·17

Hence, in this case, the differences of the errors for the several chronometers are as follows:—

		s
A	+	14·23
C	+	109·28
I	—	56·42

The interval between the epochs is 11 days. Hence, proceeding as in the preceding example, we have,—

		Errors.			Rates.
		h	m	s	s
Chron. A	—	10	1	20·98	+ 1·29
C	—	8	56	26·91	+ 9·93
I	—	11	9	38·96	— 5·13

Which errors and rates correspond to the mean epoch between Feb. $5^{d}·875$, and Feb. $16^{d}·875$ (the observations having been made at 9 A.M.), that is, to the period Feb. $11^{d}·375$.

If the errors of the chronometers on Greenwich mean time were

* It might, at first sight, seem more consistent to consider chronometer R fast on Greenwich + $5^{h}\ 11^{m}\ 52^{s}·85$; but since, as we have before observed, chronometers may be indifferently considered *fast*, or their complements to 12 hours *slow*, and *vice versâ*, and as we recommend all the chronometers to be subjected to uniformity of treatment, it is preferable in this case to consider its error *slow*, like that of the two other chronometers under discussion with it.

then required, they could be obtained, as in the preceding example, by applying the longitude in time of the spot of observation to the above errors on local mean time.

The preceding examples suppose (as will generally be the case) that both the observations for the time at the place have been made under the same meridian. It may, however, occasionally be necessary to employ observations made under two different meridians, since it may sometimes happen that a vessel may not remain sufficiently long at a given station to be able to obtain two observations at a suitable interval for the deduction of the rate; but, after obtaining one observation at the one station, may proceed to another, and obtain a second independent determination of the error on local mean time at that place. In this case, if the difference of longitude of the two stations be accurately known, the observations may be combined as before for the determination of the mean error and rate.

Example 3. By observations of the fall of the time-ball at Portsmouth Dockyard, on May 3d, 1854, the errors of the chronometers on Greenwich mean time at 1 P.M. of a vessel lying at Spithead were as under,—

		h	m	s
D	+	0	1	13·50
M	−	0	4	17·90
P	−	0	11	58·00

On the vessel's arrival at Falmouth, on May 9th, single-altitude observations at $9^h\ 13^m$ A.M., gave the errors on local mean time as follows:—

		h	m	s
D	+	0	21	38·40
M	+	0	15	29·40
P	+	0	8	48·90

From these observations the respective errors and rates are required.

The difference of longitude between Greenwich and Falmouth (Pendennis Castle) is $+\ 0^h\ 20^m\ 10^s{\cdot}80$. Hence, by the observations at Portsmouth on May 3d, the errors of the chronometers on Falmouth mean time were,—

		h	m	s
D	+	0	21	24·30
M	+	0	15	52·90
P	+	0	8	12·80

Comparing these with the Falmouth observations on May 9th, it appears that the differences were,—

$$\begin{array}{l} \quad\quad\quad\ \ \text{s} \\ \text{D} + 14{\cdot}10 \\ \text{M} - 23{\cdot}50 \\ \text{P} + 36{\cdot}10 \end{array}$$

Also the interval between May $3^{d}{\cdot}042$ Greenwich mean time, and May $8^{d}{\cdot}884$ Falmouth mean time, is $5^{d}{\cdot}856$ (allowing a correction of $0^{d}{\cdot}014$ for the difference of longitude).

Hence the errors of the chronometers on Falmouth mean time at the mean epoch, May $5^{d}{\cdot}956$,* and the corresponding rates, were,—

	Errors.			Rates.
	h	m	s	s
D	+ 0	21	31·35	+ 2·41
M	+ 0	15	41·15	− 4·01
P	+ 0	8	30·85	+ 6·16

This example affords an instance in which, in default of duplicate observations of a similar character, the errors and rates are of necessity deduced from independent observations of different kinds: it also affords an illustration of an important remark of Raper's, "that as the longitudes of the several places approach to precision, ships will employ the difference of longitude as a means of obtaining directly the *sea rates* of their chronometers, instead of waiting to obtain harbour rates;

* In this and the preceding examples, as will often happen in practice, the date, corresponding to the mean epoch of the errors and rates, is not an exact or integral date. This will be of little consequence in the measurement of meridian distances, because when the intervals which enter into the equations are accurately expressed, they are almost certain to involve fractional quantities, even when the observations are referred to mean noon, owing to the correction to the intervals, for difference of longitude.

If, however, this be thought troublesome in ordinary practice, the errors may be brought up to any convenient exact integral date, by applying to them a proportional part of the mean daily rate, due to the interval between the actual and selected epochs.

Thus, in the last example, suppose it was proposed to express the errors for the exact date, May $6^{d}{\cdot}0$. Here the interval between the actual and selected epochs is $0^{d}{\cdot}044$, and the changes of rate for that interval, for the three chronometers respectively, $+ 0^{s}{\cdot}11$, $- 0^{s}{\cdot}18$, and $+ 0^{s}{\cdot}27$.

Hence applying these several corrections to the above errors, we have for the new errors at the epoch, May $6^{d}{\cdot}0$,—

	h	m	s
D	+ 0	21	31·46
M	+ 0	15	40·97
P	+ 0	8	31·12

and so on in similar cases.

thus exemplifying one of the most important ends to which the perfection of hydrography can serve."*

The above examples will probably be amply sufficient in illustration of the general mode of proceeding; a few practical observations on the subject suggest themselves in continuation.

I. When the time has been obtained by "equal altitudes," the epochs to which the observations refer are the instants of apparent noon. Hence, since the "equation of time" is ever varying, in strictness they should be reduced to their corresponding moments of mean time, in order that the interval between any two successive epochs, and also the mean epoch, may be correctly expressed.

In the majority of cases in practice, this degree of refinement will, doubtless, scarcely be necessary; but in cases where the chronometers had large rates, and when it is wished to proceed with every regard to precision, these minutiæ should be attended to.

II. For the convenience of correctly describing moments of time involving fractional portions of a day, a table is given in the Appendix for converting intervals of time (or longitude) into their equivalent fractions of a day. The table will be found useful in the above instances, and also in facilitating the accurate expression of intervals, in cases where the time has been obtained by single altitudes taken, as will frequently be the case, at *about*, but not exactly, the same time of day.

It will be sufficient to express the intervals to three places of decimals; no inconvenience can attend this nicety, since, doubtless, the arithmetic processes involved in the manipulation of the corrections for the rates of the chronometers will usually be performed by logarithms.

III. The consideration as to what may be the best interval for determining the rates of chronometers is a question of some interest. If the rate of a chronometer were deduced from the comparison of two consecutive values of its error on mean time, taken at an interval of one day, the rate so obtained would be vitiated by the whole amount of the algebraic difference of any errors which might exist in the determination of the two errors on time at the place. If the rate depended on observations made at an interval of n days, the rate would be affected in like

* "Nautical Magazine" for 1839, p. 406.

manner by the *n*th part of that difference. Hence, if the stability of rate of the chronometer could be implicitly relied on, its value would be determined with more and more exactness as the interval between the observations of *n* days increased in amount.

In practice, however, this theoretic view is limited in its application, by the impossibility of depending confidently on the steadiness of the rate over long periods, and by the consequent necessity for checking the performances of chronometers by frequent determinations of their errors, and thus breaking up the intervals on which the rates depend into short periods.

As a matter of practice, therefore, it seems advisable, when circumstances permit, that the rate of a chronometer should not depend on observations made at an interval of *less* than *five* or *more* than *ten* days. Seven days will be found a convenient average interval, and in the case of eight-day chronometers, moreover, it embraces the period affected by the whole length of the chain. With the above limitations, it may be laid down as a maxim, that "chronometers cannot be rated too often when time and opportunity permit."

IV. It seems advisable, moreover, when the measurement of meridian distances is in contemplation, that, in so far as may be practicable, the two rates employed should depend on observations made at equal intervals of time; since when the intervals are very unequal, the small errors of observation do not exercise an equal influence on the final results, and their values are unduly affected by the errors of observation attendant on the rate determined at the shorter of the two periods. It is of importance, moreover, that the observations for the determination of the errors and rates should, whenever practicable, be of a *similar* character, as previously suggested (*ante*, p. 68). The method of "equal altitudes" is strongly recommended for adoption, as being the most correct one at the disposal of the seaman.

V. Since it is expedient that those who may be desirous of undertaking the deduction of meridian distances should be ready to avail themselves of every opportunity that circumstances may afford, we would recommend, that in cases where a ship is lying in port, and the period of her departure uncertain, observations for the errors of the chronometers on time should be made once every week, but not necessarily with any intention

of immediate or even subsequent reduction, unless actually needed. By this plan, whenever the departure took place, however unexpectedly, the data from the two last observations would be immediately available for the determination of a working error and rate, even if time did not permit additional observations to be taken immediately before starting for the voyage. Of course, on return into port at the termination of a cruise, observations for re-determining the errors and rates should be made at the earliest convenient opportunity.

CHAPTER V.

ON RATING CHRONOMETERS, AND ON THE DETERMINATION OF THEIR ERRORS ON LOCAL MEAN TIME, BY THE COMBINATION OF SEVERAL OBSERVATIONS OF A SIMILAR CHARACTER TAKEN WITHIN ANY CONVENIENT INTERVAL.

WE shall now proceed to show how any number of similar observations can be combined for the purpose of ascertaining the error and rate.

Proceeding on the same principle which guided us before, when treating of the simple method of rating by duplicate observations, viz. that equal confidence may be placed in all the observations (if not, the doubtful ones should be rejected), the determination of the error* simply consists, in general, in taking the arithmetic mean of the several errors corresponding to the respective times of observation, which will manifestly give us the mean error corresponding to the epoch indicated by the mean of those several times.

It may happen that this epoch may not be an integral or exact date, and might, therefore, not be considered altogether suited for practical manipulation; as a remedy for this possible objection we shall take occasion to show, further on, that the original error thus found can subsequently be easily reduced to any required and convenient epoch.

In order to enable us to treat the observations for rate in a systematic manner, and to deal with them in accordance with mathematical principles, it is necessary to assume two postulates: first, that equal confidence can be placed in all the observations (if not, those of doubtful character should be rejected); and secondly, that if any change of rate is taking place, and the rate be not constant, then that the change of rate is progressing uniformly and in proportion to the time.

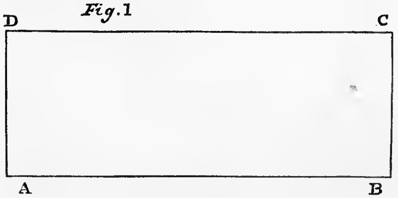
Of course, it is not pretended that this is ever rigorously

* See remarks on Example IV. p. 97.

true, except by mere accident; all we contend for is, that the assumption of a progressive and uniform acceleration of rate (whether in a gaining or losing direction) is admissible during short periods, and that it is highly probable that, in the long run, the actual inequalities of the rate are compensated for, and its mean condition not unsatisfactorily represented.

If the rate of a chronometer were uniform, the accumulation of its rate during any given time could be graphically represented by the area of a rectangle, whose base, A B, or t, should represent the time elapsed, and altitude B C, or a, the uniform rate.

Fig. 1

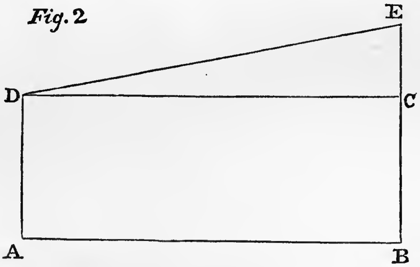


$$A\,B = t$$
$$B\,C = a$$

Then, Area of rectangle, or accumulated rate $= t\ a$.

If it be assumed that at the commencement of the period, t, the rate was a, but that at its termination the rate had altered to $a + b$,* the change of rate from a to $a + b$ having been equable and uniformly proportional to the time; then the whole accumulation during the period t could be represented by the area of the figure A B E D.

Fig. 2



$$A\,B = t$$
$$B\,C = a$$
$$C\,E = b$$
$$B\,E = a + b$$

That is, by the area of the rectangle A B C D added to the area of the triangle D C E.

* The algebraic signs of a and b being positive or negative, as the case may be; *gaining* rates, and the alterations of rates in a *gaining direction*, being considered *positive*, and *losing* rates, and the alterations of rates in a *losing direction*, being held to be *negative*.

Hence,

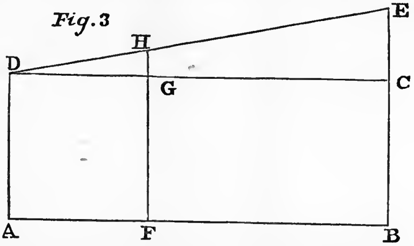
$$\text{Area of whole figure, or accumulated rate} = ta + t\frac{b}{2}$$

Now let A D or B C $= x$ (fig. 3) be the rate of a chronometer at any given epoch, and after the lapse of m days, let the rate be supposed to have changed to B E $= x + y$; then the whole accumulation of rate during the period of m days will be represented by the expression,

$$\text{Whole accumulation} = m\,x + \frac{m}{2} \cdot y \qquad (1)$$

which represents the area of the whole figure A B E D.

Fig. 3



A B $= m$
A F $= n$
B C $= x$
C E $= y$
B E $= x + y$

Similarly, the accumulation for any partial interval, after the lapse of n days, if A F or D G $= n$, will be represented by the area A F H D, or by the sum of the areas A F G D and D G H.

Now $\qquad$ D G : G H :: D C : C E

or $\qquad n$: G H :: m : y

$$\therefore \text{G H} = \frac{n}{m} \cdot y$$

Hence,

$$\text{Area D G H} = \frac{n}{2} \cdot \text{G H}$$

$$= \frac{n^2}{2\,m} \cdot y$$

and whole accumulation of rate for any partial interval, n,

$$= n\,x + \frac{n^2}{2\,m} \cdot y \qquad (2)$$

The rate itself at the time being represented by F H,

$$= x + \frac{n}{m} \cdot y \qquad (3)$$

Now suppose that observations for the errors of a chronometer on local mean time were made on several days consecutively, or nearly so—say on the 9th, 11th, 12th, 14th, 17th, and 18th of any given month—and that we assume, that although doubtless affected by errors of observation, yet that equal confidence can be placed on all of them; then, by taking the difference between the errors on the 9th and 11th, 9th and 12th, 9th and 14th, and so on, we should have a series of quantities, a, b, c . . . &c., representing the accumulations of the rate, as given by observation, for the several partial intervals between the first and each subsequent day's observation in succession; and $n_1, n_2, n_3 \ldots m$, representing the several partial intervals, we should have, on the principle of formula (2), a series of equations of condition of the following form:—

$$n_1 x + \frac{n_1^2}{2m} y = a$$

$$n_2 x + \frac{n_2^2}{2m} y = b$$

$$n_3 x + \frac{n_3^2}{2m} y = c$$

$$\ldots\ldots = \ldots$$

$$m x + \frac{m^2}{2m} y = e$$

If the observations from which these equations are obtained were absolutely *correct*, the values of the unknown quantities deduced from some of the equations ought, when substituted, to satisfy the remainder; but as this can never be the case in actual practice, it is advisable to combine the equations so as to give the most probable values of the unknown quantities, and the *Method of Least Squares*, well known to astronomers, seems best adapted for that purpose.

The mode of proceeding in the case before us is as follows:—

Taking the sum of the above partial equations, we have

$$(n_1 + n_2 + n_3 + \ldots + m)x + \frac{n_1^2 + n_2^2 + n_3^2 + \ \ldots\ + m^2}{2m} y = a + b + c + \ldots + e,$$

which may be put under the form

$$\mathrm{A}x + \mathrm{B}y = \mathrm{P} \qquad (4)$$

and which gives us our first final equation.

Again, multiplying each of the original partial equations by the coefficient of x in it, we obtain another set of equations, as follows:—

$$n_1^2 x + \frac{n_1^3}{2m} y = n_1 a$$

$$n_2^2 x + \frac{n_2^3}{2m} y = n_2 b$$

$$n_3^2 x + \frac{n_3^3}{2m} y = n_3 c$$

$$. \quad . \quad . \quad . \quad . = . \quad . \quad .$$

$$m^2 x + \frac{m^3}{2m} y = m e$$

Taking their sum,

$$(n_1^2 + n_2^2 + n_3^2 + \ldots + m^2)\, x + \frac{n_1^3 + n_2^3 + n_3^3 + \ldots + m^3}{2m} y$$
$$= n_1 a + n_2 b + n_3 c + \ldots + m e$$

which may be put under the form,

$$C x + D y = Q \qquad (5)$$

and which gives us our second final equation.

From these two equations (4) and (5) we proceed to determine x and y.

From (4),

$$y = \frac{P - A x}{B}$$

From (5),

$$y = \frac{Q - C x}{D}$$

Therefore, equating the values of y,

$$\frac{P - A x}{B} = \frac{Q - C x}{D}$$

Hence,

$$D P - A D x = B Q - B C x$$

and

$$(A D - B C) x = D P - B Q$$

$$\therefore x = \frac{D P - B Q}{A D - B C} \qquad (6)$$

x being determined, y becomes known from (4), since

$$y = \frac{P - A x}{B}$$

These formulæ are more simple than they at first sight appear, and will offer but little difficulty in their numerical solution.

For, on examination, it will appear that

$$A = n_1 + n_2 + n_3 + \ldots\ldots + m$$

= the sum of the several partial intervals between the observations.

In practice, these numbers will usually be integers, or, at any rate, it would be easy to arrange that they should be so, by taking the observations accordingly; or by reducing them to corresponding and regular epochs by approximate corrections when necessary.

Again,

$$C = n_1^2 + n_2^2 + n_3^2 + \ldots\ldots + m^2$$

= the sum of the squares of the several partial intervals.

Also,

B = the sum of the squares divided by $2\,m$; and D = the sum of their cubes divided by $2\,m$.

Likewise,

P = the sum of all the partial differences (a, b, c . . . . e) between the computed errors of the chronometers on time at the place.

And,

Q = the sum of the same quantities multiplied respectively by the coefficients of x (viz. n_1, n_2, n_3 . . . . m) in the first partial equations.

It is also worthy of note, that if more than one chronometer is employed, and the rates of several are being determined at the same time, by the same series of observations, which will usually be the case in actual practice, the coefficients A, B, C, and D, and hence also the factor $\frac{1}{AD - BC}$, quantities depending on the periods between the observations, will be constants for all the chronometers. The only variable quantities will be the numbers P and Q, which will be different for each chronometer.

We shall proceed to illustrate these formulæ by some practical examples.

Example, No. I.

By equal altitudes the errors of a chronometer, on local mean time at noon, were found to be as follows:—

		h	m	s
May 3d	Chron. fast	3	17	4·55
5th		3	17	14·00
8th		3	17	24·90
9th		3	17	29·25
12th		3	17	41·40
14th		3	17	49·50

Differences.

	s
$a =$	+ 9·45
$b =$	+ 20·35
$c =$	+ 24·70
$d =$	+ 36·85
$e =$	+ 44·95
Sum =	+ 136·30

Here,

		Squares.	Cubes.
$n_1 =$	2	4	8
$n_2 =$	5	25	125
$n_3 =$	6	36	216
$n_4 =$	9	81	729
$m =$	11	121	1331
Sum	33	267	2409

Also,

	s
$n_1 a =$	+ 18·90
$n_2 b =$	+ 101·75
$n_3 c =$	+ 148·20
$n_4 d =$	+ 331·65
$m\ e =$	+ 494·45
	+ 1094·95

Whence we have

$$A = 33$$

$$B = \frac{267}{22} = 12{\cdot}136 \qquad P = +\ 136{\cdot}30$$

$$C = 267 \qquad Q = +1094{\cdot}95$$

$$D = \frac{2409}{22} = 109{\cdot}5$$

Now, by formula (6),

$$x = \frac{D\,P - B\,Q}{A\,D - B\,C}$$

Therefore, substituting numerical values as above,

$$x = \frac{14924{\cdot}85 - 13288{\cdot}27}{3613{\cdot}5 - 3240{\cdot}3}$$

$$= \frac{1636{\cdot}58}{373{\cdot}2}$$

$$= 4^{s}{\cdot}385$$

Also,
$$y = \frac{P - Ax}{B}$$
$$= \frac{136{\cdot}30 - 144{\cdot}705}{12{\cdot}136}$$
$$= \frac{-8{\cdot}405}{12{\cdot}136}$$
$$= -0^s{\cdot}693$$

These values of x and y thus obtained are dependent on all the observations made during the period between the 3d and 14th, and from them the state of the rate at any moment during that period can be determined.

Thus, on May 3d, at the moment when the first observation was made,

$$\text{The rate } x = +\ 4^s{\cdot}385$$

And on May 14th, at the moment when the last observation was taken,

$$\text{The rate } x + y = +\ 4^s{\cdot}385 + (-0^s{\cdot}693)$$
$$= 3^s{\cdot}692$$

And at any other time, May $(3+n)^{\text{days}}$, by formula (3),

$$\text{The rate } = x + \frac{n}{m} \cdot y$$

$$= +\ 4^s{\cdot}385 - \frac{n}{m} \cdot 0^s{\cdot}693$$

In order to determine the error of the chronometer, since we assume that equal confidence can be placed on all the observations (or otherwise they would have been rejected), we have, by taking the arithmetic mean between all the errors, the mean error corresponding to the mean of the times as a mean epoch.

Hence we find in this case,

May $8^d{\cdot}5$, Chron. fast $3^h\ 17^m\ 27^s{\cdot}27$

To find the rate corresponding to this epoch, since May $8^d{\cdot}5$ is $5^d{\cdot}5$ in advance of May 3d, $n = 5{\cdot}5$, and by our formula (3),

$$\text{Rate} = x + \frac{n}{m} \cdot y$$

$$= x + \frac{5{\cdot}5}{11} \cdot y$$
$$= 4^s{\cdot}385 - \tfrac{1}{2}\,(0^s{\cdot}693)$$
$$= 4^s{\cdot}039$$

If it should be thought inconvenient that the mean arithmetic epoch should involve a fractional expression, and an epoch corresponding to an integral number of days, or an exact date should be preferred, the correction due to the mean arithmetic error can easily be determined.

First, let the assumed integral epoch, to which it is proposed to reduce the error and rate, be in advance of the mean arithmetic epoch.

Let n = the interval between the period of the first observation and the mean arithmetic epoch; and n' = the interval between the period of the first observation and the proposed epoch.

Then, by formula (2),

$n\,x + \frac{n^2}{2\,m}\,y =$ accumulation of rate to mean arithmetic epoch.

And

$n'x + \frac{n'^2}{2\,m}\,y =$ accumulation of rate to proposed integral epoch.

Then, obviously,

$$(n' - n)\,x + \frac{n'^2 - n^2}{2\,m} \cdot y$$

or

$$(n' - n)\,x + \frac{\overline{n' + n} \,.\, \overline{n' - n}}{2\,m} \cdot y \qquad (7)$$

will be the correction to the mean arithmetic error.

Again, if n' is less than n; that is, if the proposed integral epoch is prior to the mean arithmetic epoch, then the correction will obviously be,

$$\overline{n - n'}\,x + \frac{\overline{n + n'} \,.\, \overline{n - n'}}{2\,m}\,y \qquad (8)$$

For example, in the case before us, the mean arithmetic epoch is May $8^{d}\cdot5$. Let it be supposed that for this we propose to substitute the exact epoch, May 9^{d}.

Here $n' = 6$ and $n = 5\cdot5$;

therefore, formula (7),

$$(n' - n)\,x + \frac{\overline{n' + n} \,.\, \overline{n' - n}}{2\,m} \cdot y$$

becomes

$$0\cdot5 \times 4^{s}\cdot385 - \frac{11\cdot5 \,.\, 0\cdot5}{22} \,.\, 0^{s}\cdot693$$

or

$$2^{s}\cdot192 - 0^{s}\cdot181 = 2^{s}\cdot011$$

Hence, at the epoch, May 9^d, the chronometer is fast on local mean time,

$$3^h\ 17^m\ 27^s{\cdot}27 + 2^s{\cdot}01, \quad \text{or } 3^h\ 17^m\ 29^s{\cdot}28$$

Also the corresponding rate at this time, by formula (3),

$$= x + \frac{n'}{m} \cdot y$$
$$= 4^s{\cdot}385 - \frac{6}{11} \cdot 0^s{\cdot}693$$
$$= 4^s{\cdot}007$$

Again, let the proposed integral epoch be May 8^d, a date in the rear of the mean arithmetic epoch.

Here $\qquad n' = 5 \quad \text{and} \quad n = 5{\cdot}5\,;$

therefore the expression from formula (8),

$$\overline{n - n'}\, x + \frac{\overline{n + n'}\,.\,\overline{n - n'}}{2\,m}\, y$$

becomes $\qquad 0{\cdot}5 \times 4^s{\cdot}385 - \frac{10{\cdot}5\,.\,0{\cdot}5}{22}\, 0^s{\cdot}693$

or $\qquad 2^s{\cdot}192 - 0^s{\cdot}165 = 2^s{\cdot}027$

Hence, at the epoch May 8^d, the chronometer is fast on local mean time $3^h\ 17^m\ 27^s{\cdot}27 - 2^s{\cdot}027$, or $3^h\ 17^m\ 25^s{\cdot}24$.

And the corresponding rate at this time by formula (3)

$$= x + \frac{n'}{m} \cdot y$$
$$= 4^s{\cdot}385 - \frac{5}{11}(0^s{\cdot}693)$$
$$= 4^s{\cdot}070$$

In selecting a second example, we shall take one from M. Daussy's* paper on rating chronometers, a translation of which is given in the "Nautical Magazine," vol. iii. for 1834, p. 393.

* "Méthode de Calcul pour obtenir la Marche des Chronomètres. Par M. Daussy. Extrait de la Connoissance des Tems de 1835." This paper is reprinted in the "Recherches Chronomètriques," p. 41.

Example, No. II.

By single altitudes P.M. taken on the 11th July, and on several subsequent days, it was found that the errors of a chronometer on local mean time were as follows:—

		h m s		h m s
July 11th	at	3 1 22	Chron. slow	6 41 32
12th		3 1 44		6 41 32
14th		2 33 31		6 41 34
16th		2 50 40		6 41 42
18th		3 13 52		6 41 51
19th		2 47 3		6 41 50

As these observations were not all made at exactly the same time in the day, the intervals between them would not be exact integers; to remedy this inconvenience, it would be advisable to reduce the observations by means of an approximate rate to what they would have been had they all been taken at, say 3 P.M.

Comparing the first and last observations on the 11th and 19th, we have for the approximate daily rate $\frac{18}{8} = 2^s{\cdot}25$.

Correcting the observations by means of this rate, we have,

		h m		h m s
July 11th	at	3 0 P.M.	Chron. slow	6 41 32·00
12th				6 41 32·00
14th				6 41 34·04
16th				6 41 42·01
18th				6 41 50·98
19th				6 41 50·02

And this operation, be it observed, is an illustration of what we remarked before (p. 87), viz. that if the intervals between the observations were not integral numbers, it would be easy to arrange that they should be so; and although, in the instance before us, the corrections to the observations are very minute, yet, if the rates were large, they could not with propriety be neglected.

On examining these observations, it appears that

		Squares.	Cubes.			
$n_1 =$	1	1	1	$a = -$	$0^s{\cdot}00$	$n_1\, a = -\ 0^s{\cdot}00$
$n_2 =$	3	9	27	$b = -$	$2{\cdot}04$	$n_2\, b = -\ 6{\cdot}12$
$n_3 =$	5	25	125	$c = -$	$10{\cdot}01$	$n_3\, c = -\ 50{\cdot}05$
$n_4 =$	7	49	343	$d = -$	$18{\cdot}98$	$n_4\, d = -\ 132{\cdot}86$
$m =$	8	64	512	$e = -$	$18{\cdot}02$	$m\, e = -\ 144{\cdot}16$
Sum	24	148	1008		$-\ 49{\cdot}05$	$-\ 333{\cdot}19$

Whence we have,

$$A = 24$$

$$B = \frac{148}{16} = 9{\cdot}25 \qquad\qquad P = -\ 49{\cdot}05$$

$$C = 148 \qquad\qquad Q = -\ 333{\cdot}19$$

$$D = \frac{1008}{16} = 63$$

And by formula (6),

$$x = \frac{\text{D P} - \text{B Q}}{\text{A D} - \text{B C}}$$

And substituting numerical values,

$$x = \frac{63 \times (-\ 49{\cdot}05) - 9{\cdot}25\ (-\ 333{\cdot}19)}{24 \times 63 - 9{\cdot}25 \times 148}$$

$$= \frac{-\ 3090{\cdot}15 - (-\ 3082{\cdot}00)}{1512 - 1369}$$

$$= \frac{-\ 8{\cdot}15}{143} = -\ 0^{s}{\cdot}057$$

Also, by formula (4),

$$y = \frac{\text{P} - \text{A}\,x}{\text{B}}$$

$$= \frac{-\ 49{\cdot}05 - 24\,(-\ 0{\cdot}057)}{9{\cdot}25}$$

$$= \frac{-\ 49{\cdot}05 - (-\ 1{\cdot}368)}{9{\cdot}25}$$

$$= \frac{-\ 47{\cdot}682}{9{\cdot}25} = -\ 5^{s}{\cdot}154$$

x and y being thus determined, the state of the error and rate at any time during the period of the observations becomes known by the application of our formulæ (3), (7), and (8).

By taking the mean of all the errors we have, at the mean epoch, July $15^{d}\cdot125$, (or July 15th, 3 P.M.), chronometer slow $6^{h}\,41^{m}\,40^{s}\cdot17$; and by formula (3), the rate at this time,

$$= x + \frac{n}{m} \cdot y$$
$$= -\ 0^{s}\cdot057 - \tfrac{1}{2}\,(5^{s}\cdot154)$$
$$= -\ 2^{s}\cdot634$$

The error and rate* thus found are as good mean values as the data employed can afford. If it be thought that the values of x and y seem inconsistent, it must be remembered that the actual observations, as will appear on inspecting them, do not offer regular results; whence it is to be inferred, either that the observations themselves were not good, or that the chronometer was really irregular in its rate.

As objections may be raised to all theories, it is not improbable that it may be advanced against the propriety of the leading idea which distinguishes our present mode of proceeding, that when we speak of the rate's altering from its original value of x to $x + y$, it by no means follows that any alteration really takes place, and that x may be uniform; to which we reply, that supposing that to be the case, the formula will prove its truth by giving a value of y equal, or nearly equal, to 0.

The following example will illustrate this:—

Example, No. III.

			h	m	s
June	3d	Chron. slow	1	10	50
	4th		1	10	50·3
	6th		1	10	50·7
	7th		1	10	50·8
	9th		1	10	51·1
	11th		1	10	51·7
	13th		1	10	52·2

* Comparing the above result with that given in M. Daussy's paper, it will be found that, employing the results there given, and reducing the errors and rates to the same epoch as above, July 15th, we shall obtain identical results. (See "Naut. Mag." 1834, p. 395, or "Rech. Chron." p. 48.)

Hence we have,

	Squares.	Cubes.		
			s	s
$n_1 = 1$	1	1	$a = -0.3$	$n_1 a - 0.3$
$n_2 = 3$	9	27	$b = -0.7$	$n_2 b - 2.1$
$n_3 = 4$	16	64	$c = -0.8$	$n_3 c - 3.2$
$n_4 = 6$	36	216	$d = -1.1$	$n_4 d - 6.6$
$n_5 = 8$	64	512	$e = -1.7$	$n_5 e - 13.6$
$m = 10$	100	1000	$f = -2.2$	$mf - 22.0$
Sum 32	226	1820	-6.8	-47.8

Whence we obtain,

$$A = 32 \qquad\qquad P = -6.8$$
$$B = \frac{226}{20} = 11.3 \qquad\qquad Q = -47.8$$
$$C = 226$$
$$D = \frac{1820}{20} = 91$$

And by formula (6),

$$x = \frac{DP - BQ}{AD - BC}$$

And substituting numerical values, it will be found that

$$x = -0^s.219$$

Again,
$$y = \frac{P - Ax}{B}$$
$$= 0^s.018$$

Which results, on examination, will appear very consistent with the data employed.

Also, at the mean arithmetic epoch, June $7^d.57$, the chronometer was slow $1^h\ 10^m\ 50^s.97$; and by formula (3), the rate at this time,

$$= x + \frac{n}{m} y$$
$$= -0^s.219 + \frac{4.57}{20} . 0^s.018$$
$$= -0^s.215$$

And reducing these results to the mean integral epoch, June 8^d, we shall have, by formulæ (7) and (3),

		h	m	s
June 8^d	Chron. slow	1	10	51·06
	Daily rate			— 0·21

We shall conclude these examples with one purposely selected, on account of the apparent fluctuations in the rate, in order to show that this circumstance will not prevent our formula from doing itself justice, and from exhibiting probably true values of x and y.

Example, No. IV.

			h	m	s
Nov.	13th	Chron. fast	1	0	4·4
	15th		1	0	7·7
	17th		1	0	8·9
	20th		1	0	6·5
	22d		1	0	4·3
	23d		1	0	2·1

Here,

	Squares.	Cubes.	s	s
$n_1 = 2$	4	8	$a = + 3·3$	$n_1\, a = + 6·6$
$n_2 = 4$	16	64	$b = + 4·5$	$n_2\, b = + 18·0$
$n_3 = 7$	49	343	$c = + 2·1$	$n_3\, c = + 14·7$
$n_4 = 9$	81	729	$d = - 0·1$	$n_4\, d = - 0·9$
$m = 10$	100	1000	$e = - 2·3$	$m\ e = - 23·0$
Sum 32	250	2144	+9·9	+39·3
			−2·4	−23·9
			+7·5	+15·4

Whence we have,

$$A = 32$$

$$B = \frac{250}{20} = 12·5 \qquad P = + 7·5$$

$$C = 250 \qquad Q = + 15·4$$

$$D = \frac{2144}{20} = 107·2$$

And by formula (6),

$$x = \frac{DP - BQ}{AD - BC}$$

And substituting numerical values,

$$x = 2^{s}\cdot 002$$

Also, by formula (4),

$$y = \frac{P - Ax}{B}$$
$$= -\ 4^{s}\cdot 525$$

From the values of x and y thus found the state of the rate at any moment can be determined.

On examining these values of x and y, it appears that at the moment of the first observation the chronometer had a gaining rate of $2^{s}\cdot 002$, and at the moment of the last observation a losing rate $(x + y)$ of $2^{s}\cdot 523$; and our hypothesis being that the change of rate has taken place uniformly, it follows that the rate of the chronometer first continually decreased till it came to zero, and then continued to increase its losing rate till it attained its final amount; and it follows, also, that the chronometer must have passed through a period when its error was a maximum. An examination of the data of the observations will show that both these suppositions are perfectly consistent with the apparent facts of the case. If, in order to determine the error of the chronometer on local mean time, we were to proceed as usual, and find the mean error corresponding to the mean arithmetic epoch, we should have

Nov. $18^{d}\cdot 33$ Chron. fast $1^{h}\ 0^{m}\ 5^{s}\cdot 65$

Such a result, however, on an inspection of the observations, could not be considered satisfactory, unless we are prepared to admit great irregularities in the results of the daily observations themselves. If, on the contrary, as is more probable, the daily rate of the chronometer itself had really fluctuated, and we, having confidence in the observations, agree to assume that position as true, then it will be proper to treat the determination of the error somewhat differently, and after ascertaining its successive values at the epoch of the final observation, from each day's observation taken separately, then, as a final *working* error, to take a mean of all the separate results so determined. The computation of these several errors can be made without difficulty from an equation analogous in form to formulæ (7) and (8). The correction to the first day's observation

will evidently be the whole accumulation of the rate, during the whole period between the first and last observations, which is given by formula (1),

$$m\,x + \frac{m}{2}\,y$$

That for the second day will be the whole accumulation diminished by that due to the partial period, n_1; that is,

$$m\,x + \frac{m}{2}\,y - \left\{n_1\,x + \frac{n_1^2}{2\,m}\cdot y\right\}$$

$$= m\,x + \frac{m^2}{2\,m}\,y - \left\{n_1\,x + \frac{n_1^2}{2\,m}\cdot y\right\}$$

$$= \overline{m - n_1}\,x + \frac{m^2 - n_1^2}{2\,m}\cdot y$$

$$= \overline{m - n_1}\,x + \frac{\overline{m + n_1}\,.\,\overline{m - n_1}}{2\,m}\cdot y \qquad (9)$$

Those for the subsequent observations being obtained in a similar manner, by substituting in (9), in lieu of n_1, the successive values n_2, n_3, ... &c.

Applying formula (9) to the example before us, we shall have for the error of the chronometer by each day's observation, reduced to the period of the final observation on Nov. 23d, results as follows:—

			h	m	s		h	m	s
By the observations on	Nov. 13th	Chron. fast on Nov. 23d	1	0	4·40 — 2·62	=	1	0	1·78
	15th		1	0	7·70 — 5·70	=	1	0	2·00
	17th		1	0	8·90 — 6·99	=	1	0	1·91
	20th		1	0	6·50 — 5·54	=	1	0	0·96
	22d		1	0	4·30 — 2·30	=	1	0	2·00
	23d		1	0	2·10 —	=	1	0	2·10
Chron. fast Nov. 23d, Mean Error							1	0	1·79

Hence, by a combination of all the observations from the 13th to the 23d, the error of the chronometer at the epoch

$$\text{Nov. } 23^{\text{d}}, \text{ is } 1^{\text{h}}\ 0^{\text{m}}\ 1^{\text{s}}{\cdot}79 \text{ fast.}$$

And since the rate at this time by our general theory is $x + y$,

$$\text{The rate} = 2^{\text{s}}{\cdot}002 + (-4^{\text{s}}{\cdot}525)$$
$$= -\ 2^{\text{s}}{\cdot}523$$

Notwithstanding the apparent fluctuations in the rate of the chronometer, as exhibited by the above observations (since the chronometer would appear to have had a gaining rate at

first and a losing rate afterwards), the final results in this case, both for error and rate, may be considered as satisfactory; and although in most cases the mean arithmetic error corresponding to the mean of the times of observation may be considered as sufficiently accurate and convenient for practice, yet, in cases where an examination of the observations seems to indicate considerable instability or fluctuation of rate during the period of rating, and when the computer does not object to the additional labour involved in the latter more elaborate process, there is no doubt its results will usually be more satisfactory, and certainly more correct, while it will fully repay the extra trouble employed in its manipulation.*

* In a memoir published in the hydrographical appendix to the "Voyage de l'Astrolabe, Paris, 1843," by MM. Vincendon-Dumoulin and Coupvent Desbois, and since reprinted in the "Recherches Chronométriques," p. 177, a method is proposed, of combining any number of observations for rate, depending on the theory of probabilities.

It is sufficiently simple, and may be thus explained. Let d_1, d_2, d_3, ... be the differences between the observations for the error of a chronometer, on local mean time, combined together in pairs (which may be done till all the possible combinations are exhausted); and n_1, n_2, n_3, ... &c., the corresponding intervals between the observations. Dividing each difference by its corresponding interval, we have a value of the rate, and these rates have greater chances of probable exactness, the greater are the intervals, or the denominators of the fractions, $\frac{d_1}{n_1}$ $\frac{d_2}{n_2}$, &c. &c., which express the rate.

Applying the rules of the calculus of probabilities to determine the general mean rate during the interval, we must multiply each of these fractions by its denominator, take their sum and then divide it by the sum of the denominators, which is the same thing as assigning to each rate a value proportional to the denominator of the fraction which gives it; and which again, is the same as dividing the sum of all the partial differences, d_1, d_2, &c. &c., by the sum of all the intervals, n_1, n_2, &c. &c.

Hence, calling the mean concluded rate x

$$x = \frac{d_1 + d_2 + d_3 + \ldots .}{n_1 + n_2 + n_3 + \ldots .}$$

In practice, it would be convenient to combine the observations in pairs, thus; taking the difference between the first and the last errors, the second and last but one, third and last but two, and so on. If the number of observations were an odd number, one must be rejected. The mean rate thus obtained corresponds to the mean of the epochs of the separate partial rates.

Applying the above formula to the 1st example, given *ante*, p. 88,

$$\begin{aligned} x &= \frac{44^s{\cdot}95 + 27^s{\cdot}40 + 4^s{\cdot}35}{11 + 7 + 1} \\ &= \frac{76{\cdot}70}{19} \\ &= 4^s{\cdot}04 \end{aligned}$$

which rate corresponds to the epoch May $8^d{\cdot}5$, and agrees in this instance with that obtained by the method of least squares (*ante*, p. 89).

CHAPTER VI.

ON THE CHRONOMETRIC DETERMINATION OF MERIDIAN DISTANCES, AND ON THE METHOD OF ALLOWING FOR THE CHANGE OF RATES OF THE CHRONOMETERS, THAT MAY HAVE TAKEN PLACE IN THE INTERVALS BETWEEN THE OBSERVATIONS—METHODS OF DE CORNULIER—LIEUSSOU—MOUCHEZ—HARTNUP—TIARKS—REMARKS ON THESE METHODS.

THE "Meridian Distance," or "Difference of Longitude in Time," between any two places, is obtained chronometrically by comparing the errors on local mean time shown by a chronometer at the two places in succession; the error at the first place being corrected by the known rate of the chronometer in the interval, so as to give the state of the watch at the moment of the second observation. The errors of the chronometers being thus known simultaneously at the two places, their difference represents the "meridian distance," or "difference of longitude in time" between them.

If the rate of the chronometer at the first place has been accurately determined, and remains constant during the transit between the two places, the "meridian distance" between them deduced from the observations so compared, will be true within the limits of the correctness of the observations themselves; but as this will rarely be the case in actual practice, since the rate of the chronometer, on account of the mechanical imperfections in its construction, the effects of varying temperature, and other causes, is probably in a state of perpetual fluctuation, it becomes a question of considerable importance, in connexion with the accurate and systematic measurement of meridian distances, to consider how the variations of rate are to be treated, so as to produce results which, at once uniform and consistent, shall at the same time be in accordance with reasonable principles, and not open to any possible objection of caprice or vagueness.

From the earliest times, since the attention of horologers was first directed to the construction of marine chronometers, so as to produce instruments capable of accurately determining the longitude at sea, and of carefully measuring the differences of longitude of remote maritime stations, the influence of varying temperature has been distinctly recognised as one of the principal causes of marked changes of rate. It was soon perceived, moreover, that with whatever care the adjustment of the compensation might have been affected by the artist, and within whatever narrow limits, the fluctuations of rate, due to changes of temperature, might be restricted, yet that the mechanical defects of the compensation would still necessitate the application of further corrections to the results obtained from the chronometers, when accurate measurements were in question. Measures were accordingly proposed to allow for the irregularities of these instruments, the adoption of an "equation of temperature" being one of the first.

When Harrison's timekeeper was sent on its second trial voyage to the West Indies in 1764, Harrison certified to the Admiralty beforehand, what the rate of his chronometer might be expected to be, at every tenth degree of the thermometric scale, from 42° to 82° of Fahrenheit. Since its first voyage in 1761–2, Harrison had re-arranged the compensation, and time not permitting some further final adjustments, he stated, that "as the inequalities are so small, I will abide by the rate of its gaining on a mean, one second a-day for the voyage." The account of the voyage goes on to state, that on his return from Barbadoes in July 1764, "the chronometer was found to have gained only 54^s in 156 days, allowing it to have gained one second a-day, being the rate by which Harrison declared he would abide. If, however, *allowance be made for the variation of the thermometer*, as stated by him before his departure, *it will be found to have lost only* 15^s." Here we have a plain acknowledgment of the utility of temperature corrections.*

Again, in Fleurieu's voyage in 1768–9,† made for the express purpose of testing at sea the performances of the marine time-

* See "Mackay on the Longitude," vol. i. p. 275.

† "Voyage fait par l'ordre du Roi, en 1768 et 1769, pour éprouver en Mer les Horloges Marines inventées par M. F. Berthoud. Paris, 1773." The expedition was under charge of M. d'Eveux de Fleurieu, who was assisted in the observations

pieces of Louis Berthoud, that artist furnished MM. Fleurieu and Pingré, who were charged with the conduct of the observations, with a table of corrections for each of the watches, in order to "estimate their rates for the different temperatures." In the appendix to the account of the Voyage (vol. ii.), M. Fleurieu gives a full explanation of all the observations taken for the purpose of verifying the performances of the watches; we find that in all cases corrections, according to the daily observed mean temperature (taken from an interpolated table deduced from the primary one furnished by Berthoud), were applied to their indications of the time. Fleurieu also strongly insists (vol. ii. p. 427) on the necessity for commanders of ships being furnished, not only with information as to the actual errors and rates of their time-keepers, but also as to "the variations their movements should experience at different temperatures." He goes on to explain how a table of corrections is to be experimentally determined by observations of the actual performance of the instrument for different degrees of heat and cold; and he further points out (p. 438) the necessity of verifying the table from time to time by fresh experiments, and gives cautions, to prevent observers attributing to the effects of temperature changes of rate, in reality due to other causes.

Borda, in his account of the voyage of the Flore in 1772,* holds the same language as Fleurieu had previously done; thus (vol. i. p. 31) he says, "Although the equation of temperature was always very inconsiderable in the watches of Leroy and Berthoud, embarked on board the Flore, we did not omit always to take account of it."

Again (vol. ii. p. 404), Borda refers, for the verification and use of marine watches, to the appendix of the voyage of Fleurieu. He had already said (vol. i. p. 326), "We will not repeat here the good instructions that M. de Fleurieu has given for the use of marine time-keepers," &c.; and, p. 358, he adds, "We will not speak of the method of determining longitudes by marine chronometers; M. de Fleurieu has already explained it with the

by M. Pingré, an astronomer of note. They visited Cadiz, the Canaries, Goree, Martinique, St. Domingo, and the Azores. The narrative of the voyage is very copious, and full of most interesting details, reflecting infinite credit on the painstaking assiduity of M. Fleurieu and his colleague.

* "Voyages en Europe, Afrique, et Amérique, en 1771-2; par le Chev. de Borda. Paris, 1778."

greatest detail, and with all the precision that we can desire; we will refer to his work."

"It is impossible," observes M. De Cornulier, "to be more explicit." "Thus Borda recommends the use of the equation of temperature, and has himself employed it. The utility of this equation made itself so much felt, in fact, on the watch No. 8 of Berthoud, that he could not avoid having regard to it. This watch, which had no sensible acceleration, experienced suddenly a change of four or five seconds, when the Flore passed from the climate of the West Indies to that of Newfoundland and Iceland; afterwards it resumed its first rate, on the return to Brest. On this occasion, Borda takes care to make the remark (vol. i. p. 323), that this rate was conformable to the table of equations that Berthoud had furnished him with."*

Notwithstanding the authority and example of these distinguished navigators, the practice of paying regard to equations of temperature seems to have gradually fallen into disuse. M. De Cornulier thus complains: "After the voyage of the Flore, they no longer gave for the watches a table of the equation of temperature—they did not even keep any longer a register of the daily temperature of the chronometers during the voyage; all the second chapter of the appendix of Fleurieu's voyage is put into oblivion, as if it had become useless in consequence of the perfection to which compensated balances had been carried. This conduct resembles entirely that of a man who denies the danger, and closes his eyes, so as not to see it, for fear of disturbing his quietude. Yet, if we examine the tables of rates of the chosen watches which have been employed in the greater part of our grand modern voyages, we shall very soon recognise that many of them have not been exactly compensated."†

English navigators appear to have been as equally unmindful of the refinements of temperature corrections, as their French contemporaries. No allusion is made to the subject in the account of Cook's voyages, nor does the matter seem to have engaged the attention of the commanders of our various scientific voyages, and hydrographic expeditions, during the present century, in any practical degree; although the failures and anomalies which sometimes presented themselves in chronometric measurements,

* "Recherches Chronométriques," p. 113.

† Ibid. p. 117.

have often been attributed to excessive or irregular fluctuations of temperature.

The causes of this neglect have probably been twofold; first, it may have been thought, that in consequence of the great mechanical improvements in the construction of chronometers, and more especially from the great success which had been attained in the adjustments of their compensations, such minute corrections were no longer needed; and that if variations of rate took place, they could on the whole, and in the long run, be sufficiently allowed for, by assuming the theory of uniform acceleration in proportion to the interval elapsed. This theory originally, it would seem suggested by Borda, appears to have been generally adopted by modern navigators. Secondly, it is also probable, that the practical minds of seamen were often inclined to attach but little value to the minute refinements of theory, which in presence of the anomalies often revealed by observation, would appear in many respects, to verge closely on empiricism and fanciful treatment. In so far as the ordinary requirements of navigation are concerned, no doubt modern chronometers, with proper attention paid to rating them from time to time, may be sufficiently depended on without regarding these refinements; but even the conduct of common voyages would probably be improved if the corrections or allowances for changes of temperature could be reduced to a system simple, convenient, and short.*

In the future conduct of hydrographic voyages, attention to these points will perhaps become indispensable, for as maritime geography has gradually advanced towards perfection, science has become more exacting in its demands, and requires that we should avail ourselves, not only of all the mechanical improvements of modern art, but also of the ingenious investigations of mathematical analysis. Stimulated by these considerations, attention has been recently again directed to this interesting subject,

* "If accidents arising from errors in the reckoning are rare (remarks De Cornulier), and if they do not always entail fatal consequences, it is because prudent navigators hold themselves on their guard against the infidelities of the chronometer: they control as much as they can their indications by lunar distances; they direct their course very much at a distance from all land: often they heave to during the night. From these exaggerated precautions result passages much longer than they ought to be. A knowledge of the equation of temperature would always permit them to navigate more freely and with as much prudence." ("Rech. Chron." p. 106.)

with the view of making chronometers still more available than heretofore for the accurate measurement of differences of longitude.

In France, MM. De Cornulier and Lieussou (Ingénieurs Hydrographes) have recently proposed algebraic formulæ for correcting the performances of chronometers, involving a consideration of the effects of temperature, and also the influence of the acceleration. M. Mouchez (a naval officer) has also written a memoir on the same subject, and suggested a method for correcting chronometric observations for the variations of temperature. In this country, Mr. Hartnup (Director of the Liverpool Observatory) has proposed to recur to the use of tabulated daily rates, corresponding to the different degrees of the thermometric scale, and obtained by observation, in lieu of a *mean* daily rate, regardless of temperature, as is at present generally employed. This system in fact is a revival of that formerly recommended by the celebrated horologist, P. Leroy; and as it requires none but the simplest calculation and is very easy in practical use, it seems well worthy of consideration. As the work, on which we are now engaged purports to lay before its readers all useful information on the subject of the management of chronometers, it would manifestly be incomplete if it omitted to introduce to the notice of the scientific student the ingenious investigations of recent writers on this interesting subject. We therefore propose briefly to explain the leading features of their methods, referring our readers for fuller information to the more copious explanations of the authors themselves.

Method of De Cornulier.

M. De Cornulier,* an officer in the French naval service, had charge of the watches on board the Allier, during a cruise in the South Seas in 1829–31; afterwards as Director of the Naval Observatory at Lorient, he had the opportunity of enlarging his

* M. De Cornulier has published, upon the calculation of the rates of chronometers, four memoirs, which have been successively inserted in the "Annales Maritimes" for the years 1831, 1832, 1842, and 1844. All these memoirs have for their principal object the consideration of the influence that changes of temperature exercise upon the rates of these instruments, and the necessity of taking account of it, concurrently with the variation known under the name of the *acceleration*, in the determination of chronometric longitudes.

In his first memoir, De Cornulier explains the principles of his theory and its application; he devotes himself in the subsequent memoirs to the development of his

experience and of carefully studying the performances of these instruments. His investigations led him to the following conclusions:—

First. "The compensation of chronometers is not generally as perfect as is supposed—too much confidence is accorded to it. In the present day it is necessary for many of them, to have recourse to an equation of temperature, as was the practice in the first voyages for the trial of marine chronometers."

Secondly. "In the correction of longitudes obtained by chronometers, it is wrong to hold entirely to the hypothesis proposed by Borda in the voyage of the Flore, and according to which every rate which has varied, would have done it by a movement uniformly accelerated or retarded; this consideration is useful, but it ought to be combined with that which precedes it."

Thirdly. "The equation of temperature, totally forgotten, contrary to all reason, is of the two corrections that which deserves most to engage our attention; it is that which represents the greatest irregularities; it may be utilised immediately in the daily practice of navigation; whilst the other, less important in itself, less regular in its effects, cannot be determined but after an accomplished voyage. It may then be employed *à posteriori* for the perfectionment of hydrography," that is—to correct the meridian distances of the previous cruise.

Finally. "The research of the equation of temperature for each watch, is one of the most essential objects, on which a nautical observer can employ himself."*

In furtherance of these views, De Cornulier proposed to treat chronometric observations in the following manner:

At a place where the mean daily temperature was a, let the mean daily rate of a chronometer $= m$.

Subsequently at another place, where the mean daily temperature was $a + b$, let the corresponding mean daily rate $= m + n$.

method, and to justifying his opinions, by supporting them by new observations that he had made, and by evidence the most suitable for corroborating them.

These memoirs have for the most part been reprinted in the "Recherches Chronométriques." ("Rech. Chron." p. 87.)

We take the opportunity of mentioning here, that in the earlier sheets of this work, De Cornulier's name has been incorrectly printed Du Cornulier, the error not being detected till the sheets had been worked off.

* "Recherches Chronométriques," p. 88.

Let the interval between the epochs of the observations for the rates be q days, and the mean daily temperature during this period $a + d$.

Also let x be the *acceleration* of the daily rate of the chronometer, arising from imperfect mechanism; and y the variation due to a change of one degree in the thermometric scale, x will follow the law of movements uniformly accelerated, and y will be proportional to the differences of temperature.

Also let the difference between the absolute errors of the watch on mean time at the first station,* by the observations at the two places, be M.

Then assuming that the change of rate n has arisen partly from the progressive influence of the acceleration, and partly from the effects of variation of temperature,

$$n = q\,x + b\,y \qquad (1)$$

$$\mathrm{M} = q\,m + q\,d\,y + q\,x\left(\frac{q+1}{2}\right) \qquad (2)\dagger$$

Hence from (1), $$y = \frac{n - q\,x}{b}$$

* M. De Cornulier writes "on mean time of the first meridian," but as it is advisable to avoid complicating the question by a reference to Greenwich or Paris time, the above is better. Then if D be the known difference of longitude of the two stations; λ and λ', the respective errors of the chronometer on mean time at the two places

$$\mathrm{D} = \lambda' - (\lambda + \mathrm{M})$$
$$\text{And } \mathrm{M} = (\lambda' - \lambda) - \mathrm{D}$$

whence M may be determined, the errors λ and λ' being found from observation, and D being assumed to be known.

In discussing M. De Cornulier's formulæ, we have thought it best to adhere throughout to his notation, for the convenience of those who might be disposed to refer to the original work. The difference of longitude, here called D, we have elsewhere called M; while the quantity he calls M, represents the whole accumulation of the actual rate of the chronometer, in the interval between the observations.

† It will be observed that M is the whole accumulation of the rate, due to the *actual performance* of the chronometer, in the interval between the two observations. If the chronometer had kept true mean time $\mathrm{M} = o$: If the original daily rate had remained constant, then $\mathrm{M} = q\,m$: If the chronometer had been affected by a change of mean temperature d degrees acting on it for a period of q days, then $\mathrm{M} = q\,m + q\,d\,y$: If in addition it were influenced by a daily acceleration x acting on it for q days, then

$$\mathrm{M} = q\,m + q\,d\,y + q\left(\frac{q+1}{2}\right)x \quad \text{as above.}$$

The effect of the acceleration increases uniformly by a quantity x from day to day. The sum of an arithmetic series of q terms, in which the first term equals the common difference $= q\,\frac{q+1}{2}$, hence this portion of the correction $= q\left(\frac{q+1}{2}\right)x$.

and from (2), $$y = \frac{M - qm - qx\left(\frac{q+1}{2}\right)}{qd}$$

whence, solving for x,

$$x = \frac{b\left(\frac{M - mq}{q}\right) - nd}{b\left(\frac{q+1}{2}\right) - qd} \qquad (3)$$

Again from (1), $$x = \frac{n - by}{q}$$

And from (2), $$x = \frac{M - qm - qdy}{q\left(\frac{q+1}{2}\right)},$$

Hence solving for y,

$$y = \frac{n\left(\frac{q+1}{2}\right) - (M - mq)}{b\left(\frac{q+1}{2}\right) - qd} \qquad (4)$$

To obtain x and y with exactness, it is requisite that the difference of meridians (D) of the places where the observations have been made, should be perfectly known, for it is from it that we deduce the difference of the absolute errors M. It is equally necessary that the difference of temperature b or d should be a little great, in order to obtain y with some precision.

The coefficient y relative to the temperature, ought to remain very nearly constant throughout the duration of the voyage, for the errors which exist in the system of compensation of a chronometer are not of a nature to modify themselves of themselves, consequently when we have once determined this coefficient, we may usefully employ it during its course.

It is not thus with the acceleration x,* proceeding from the mechanism of the watch; from the very nature of the causes which engender it, it is essentially variable; one cannot suppose

* The acceleration is always very small in watches which have recently come from the hands of the artist: when they become considerable, the watch ought to be returned to him; it no longer merits confidence. It is to the same causes which produce the acceleration, that is attributable a fact often observed; that a watch which has stopped, rarely resumes the same rate which it had before. ("Rech. Chron." p. 96.)

The opinion given above on the magnitude of the acceleration seems at first sight at variance with a previous passage from De Cornulier's memoirs, quoted in the text (*ante*, p. 21). We take it, that he means that chronometers recently cleaned and adjusted, and fresh from the hands of the artist, are some little time before their acceleration becomes steady, and that it is then generally small in amount.

it constant for the duration of a sea-passage, and we should expose ourselves to commit grave errors in wishing to determine it in advance. This element should always be concluded from the comparison of the observations at the departure with those of arrival. It would be necessary to have acquired a very decisive experience of a watch, to dare to employ in the course of one long passage, the acceleration determined in a preceding one.

The acceleration x being very variable, or at any rate having no permanent fixity, we ought not to assume it constant, but for the smallest possible intervals, especially when it is in contemplation to redetermine the value of the coefficient of temperature y. The ordinary circumstances of navigation are not always as favourable as could be desired for this research. It is rarely that one departs from a point well determined, to repair immediately, and in a short time to another place also well determined, and whose temperature differs much from that of the first. It is necessary, also, that one should make in each of them a stay sufficiently prolonged to determine there exactly the daily rate and absolute error of the watch, things both essentially necessary to deduce a satisfactory solution from the two equations that we have established above.

In many cases, it is more important for the requirements of navigation, to know the coefficient y in a manner approximate but prompt, than to have its exact value by waiting during a long period for the circumstances requisite for its rigorous determination. The knowledge of the difference of the absolute errors of the watch (on time at the meridian of the first station) is the most difficult of all to acquire. The number of points on the globe on whose longitudes we can depend with confidence, is as yet very limited; let us endeavour then to determine the coefficient y, disregarding this element, by means of the daily rates alone,

$$m;\quad m + n';\quad m + n' + n'';\quad m + n' + n'' + n''';\quad \&c.$$

If after a very short sea-passage of q' days, we should have obtained two daily rates, m and $m + n'$, for two temperatures whose difference b' is very great, we may suppose that the acceleration x' has been nothing in this short interval: hence the equation

$$n' = q'x' + b'y$$

would become

$$n' = b'y$$

whence we have $$y = \frac{n'}{b'} \tag{5}$$

That is to say, that in this first approximation all the variation of the rate might be attributed to the change of temperature alone.

Let us suppose now, that in three successive stations, separated by intervals q', q'' of moderate length, as say twenty to twenty-five days, we should have determined the daily rates, m, $m + n'$ and $m + n' + n''$ corresponding to differences of temperature b' and b'' very decided; one might admit that the watch has had but one and the same acceleration during the two partial passages, in the intervals q' and q''; that is to say, one might admit that $x' = x''$.

We should then have the two equations,

$$n' = q' x' + b' y. \quad \text{And } n'' = q'' x' + b'' y,$$

from which equating the values of x', we should obtain,

$$y = \frac{q' n'' - q'' n'}{q' b'' - q'' b'} \tag{6}$$

which value would be rigorously exact, so long as we can assume that the acceleration x' shall have been regular.

Again, if we should have observed four successive daily rates, m; $m + n'$; $m + n' + n''$; and $m + n' + n'' + n'''$.

We might suppose that the acceleration of the chronometer during the middle passage has been a mean between the accelerations which have prevailed during the two extreme ones; that is to say, that we might make,

$$x'' = \frac{x' + x'''}{2}$$

We should then have the three following equations,

$$n' = q' x' + b' y$$

$$n'' = q'' \left(\frac{x' + x'''}{2}\right) + b'' y$$

$$n''' = q''' x''' + b''' y$$

Taking in the first and third equations, the values of x' and x''', and substituting them in the second, there would then remain but the one equation,

$$n'' = \frac{q''}{2}\left(\frac{n' - b' y}{q'} + \frac{n''' - b''' y}{q'''}\right) + b'' y,$$

containing but one unknown quantity y, and from which by reduction, we obtain,

$$y = \frac{2\,q'\,q'''\,n'' - q''\,q'''\,n' - q'\,q''\,n'''}{2\,q'\,q'''\,b'' - q''\,q'''\,b' - q'\,q''\,b'''} \tag{7}$$

When we have obtained the coefficient of temperature y, we can immediately determine the successive accelerations of the watch x', x'', x''', &c., by substituting for y its value in the equations $n' = q'\,x' + b'\,y$; $n'' =$ &c. If the accelerations appertaining to each traverse are small, and if they differ but little one from another, it is a sign that the chronometer merits great confidence.

It is not probable in practice that we should ever have occasion to combine more than four sets of observations for the daily rate, nor is it, indeed, desirable to do so, as our theory presupposes that the permanency of the acceleration x is only to be assumed during short periods. If the number of observed daily rates during a cruise exceeded four, it would be more convenient to take them in pairs, and obtain a solution for y of the form of equation (6)

$$y = \frac{q'\,n'' - q''\,n'}{q'\,b'' - q''\,b'}$$

Or again, any number of rate observations might be combined, and the most probable values of x and y, prevailing during the whole period of the observations, determined by the method of least squares, after the manner already explained in dealing with the combination of observations for determining the rate (*ante*, p. 85).*

* Assuming that during a moderate interval, within which the rates have been ascertained, the value of the coefficient x, as well as y, remained constant, the several observations for rate would give us a series of equations,

$$\begin{aligned} n' &= q'\,x + b'\,y \\ n'' &= q''\,x + b''y \\ n''' &= q'''x + b'''y \\ &\text{\&c. \&c.} \end{aligned}$$

Taking their sum

$$(n' + n'' + n''' + \ldots) = (q' + q'' + q''' + \ldots.)\,x + (b' + b'' + b''' + \ldots)\,y,$$

which may be put under the form

$$\mathrm{A}\,x + \mathrm{B}\,y = \mathrm{P} \tag{1}$$

Again, multiplying each equation by the coefficient of x in it, we obtain the new equations,

$$\begin{aligned} n'\,q' &= q^{2'}\,x + b'\,q'\,y \\ n''\,q'' &= q^{2''}\,x + b''\,q''\,y \\ n'''q''' &= q^{2'''}x + b'''q'''y \\ &\text{\&c. \&c.} \end{aligned}$$

This mode of proceeding, however, assumes that the acceleration remains uniform during the whole period, and should therefore not be employed, except during moderate intervals.

We may again, in certain circumstances, determine an approximate value of the coefficient of temperature, by means of the simple differences of the absolute errors of the chronometers on time at the first station, when we have no other observations for rate but that of the departure m.

We should employ for this research the equations

Hence

$$(n'q' + n''q'' + \ldots\ldots) = (q^{2\prime} + q^{2\prime\prime} + \ldots\ldots)\,x + (b'q' + b''q'' + \ldots\ldots)\,y,$$

which may be put under the form

$$\mathrm{C}\,x + \mathrm{D}\,y = \mathrm{Q} \qquad (2)$$

From (1) $$y = \frac{\mathrm{P} - \mathrm{A}\,x}{\mathrm{B}}$$

From (2) $$y = \frac{\mathrm{Q} - \mathrm{C}\,x}{\mathrm{D}}$$

Hence equating the values of y and reducing, we obtain

$$x = \frac{\mathrm{D\,P} - \mathrm{B\,Q}}{\mathrm{A\,D} - \mathrm{B\,C}} \qquad (3)$$

and then again from (1), x being known,

$$y = \frac{\mathrm{P} - \mathrm{A}\,x}{\mathrm{B}} \qquad (4)$$

In these equations, which would offer no difficulty in their numerical solution,

$$\mathrm{A} = q' + q'' + q''' + \ldots\ldots$$

= the sum of the several partial intervals between the epochs of the rate observations.

$$\mathrm{B} = b' + b'' + b''' + \ldots\ldots$$

= the sum of the several partial differences of temperature, corresponding to the observed rates.

$$\mathrm{C} = q^{2\prime} + q^{2\prime\prime} + q^{2\prime\prime\prime} + \ldots\ldots$$

= the sum of the squares of the several partial intervals.

$$\mathrm{D} = b'q' + b''q'' + b'''q''' + \ldots\ldots$$

= the sum of the several partial differences of temperature, multiplied into their corresponding partial intervals of time.

P = the sum of the several partial differences of rate ($n' + n'' + n''' + \ldots\ldots$).

Q = the sum of the several partial differences of rate, multiplied into the corresponding partial intervals of time ($n'q' + n''q'' + n'''q''' + \ldots\ldots$).

It is also to be observed that the coefficients, A, B, C, D, and hence also the factor $\frac{1}{\mathrm{A\,D} - \mathrm{B\,C}}$, depending on the time and temperature, will be constants for all the chronometers under discussion; the quantities P and Q depending on the differences of the observed rates, being alone variable for each chronometer.

$$M' = q' m + q' d' y + q' \left(\frac{q' + 1}{2}\right) x'$$

$$M'' = q'' (m + n') + q'' d'' y + q'' \left(\frac{q'' + 1}{2}\right) x''$$

which give these differences; and assuming for the accelerations x', x'' analogous hypotheses to those already adopted in the former case, that is, considering the acceleration to have been uniform during the period, or $x' = x''$, we have from the first equation,

$$x' = \frac{M' - q' m - q' d' y}{q' \left(\frac{q' + 1}{2}\right)}$$

And from the second,

$$x' = \frac{M'' - q'' (m + n') - q'' d'' y}{q'' \left(\frac{q'' + 1}{2}\right)}$$

Equating these values of x' and reducing, we obtain,

$$y = \frac{\left\{M'' - q'' (m + n')\right\} q' \left(\frac{q' + 1}{2}\right) - \left\{M' - q' m\right\} q'' \left(\frac{q'' + 1}{2}\right)}{d'' q' q'' \left(\frac{q' + 1}{2}\right) - d' q' q'' \left(\frac{q'' + 1}{2}\right)}$$

y being known, x' can then be obtained from either of the primary equations.

If the results at which we thus arrive, do not present all the guarantees for correctness that we could desire, they are at least useful indications, and very superior to a vague appreciation.

A circumstance very propitious for determining with precision the co-efficient y, is when a ship, experiencing considerable difference of temperature, returns to the same point from which it had originally departed, because there is then no longer any uncertainty, as to the difference of the absolute errors of the watch, M,* and the problem is no longer complicated by the errors that might be introduced into it, by an erroneous assumption of the exact difference of longitude of the terminal stations. The more we should have determined the daily rates, at the

* $M = (\lambda' - \lambda) - D$

And since $D = 0$

$M = \lambda' - \lambda$

(See *ante*, p. 107.)

various places in our voyage, during this interval, and the more we should have obtained the partial accelerations x', x'', x''', &c., the better should we know our chronometer and the degree of confidence we could accord to it, in fixing definitively the respective longitudes of the hydrographic positions that we had recognised during our cruise.

It may not be out of place to remark here, in order to fix our attention on the value of the accelerations, and on the differences that we may admit among them, that if z be the error of the acceleration, and q the number of days in the passage, the error in minutes of arc, which would supervene on the resulting longitude, would have for its expression, $\frac{1}{4} z q \left(\frac{q+1}{2}\right)$: thus an error of $0^s{\cdot}01$ in the acceleration would produce one of $12'{\cdot}5$ at the end of a traverse of a hundred days; thus quantities, which would seem to be otherwise insignificant, merit great consideration when we are dealing with the acceleration. This element of irregularity ought not to exceed a small fraction of a second in a good chronometer.

The coefficients of acceleration and temperature, x and y, having been thus determined by the application, according to circumstances, of some of the methods indicated in the preceding pages, they are to be subsequently utilised and employed in the computation of the several "meridian distances," between the places called at during the voyage, in determining the corrections to the observed errors on local mean time, due to the accumulation of the rate.

Thus let λ be the error of the chronometer at the first station, and λ' at the next. Then

$$D = \lambda' - (\lambda + \text{accumulated rate in interval.})$$

But the accumulated rate between the observations

$$M = q\,m + q\,d\,y + q\,x\left(\frac{q+1}{2}\right)$$

Hence $$D = \lambda' - \left(\lambda + q\left(m + d\,y + x\frac{q+1}{2}\right)\right)$$

which expression involving the coefficients, both of acceleration and temperature, x and y, is probably more true than the one generally adopted, which assumes only a uniform or mean daily rate, during the period; provided always that the coefficients have been deter-

mined by careful observations, and the mean daily temperatures accurately noted.

Such is briefly the system of chronometric treatment proposed by M. De Cornulier, and in truth it seems well worthy of attentive consideration. Those who may wish to study his labours in greater detail, will find them more fully developed in his memoirs,* and to them we must refer our readers for further information.

Method of Lieussou.

A very instructive and interesting memoir,† on the subject of chronometers, and on the causes of the variation of their rates, has recently been published in France, and submitted to the "Bureau des Longitudes."

The author, M. Aristide Lieussou, availing himself of the data relative to the performances of chronometers, obtained from the public trials, made at the observatories at Greenwich and Paris, has arrived at some highly curious and interesting results. By an ingenious system of graphic projection, exhibiting the connexion between the observed daily rates, the corresponding temperatures, and the time elapsed since a given epoch, M. Lieussou has investigated the laws which govern the changes to which the rates of chronometers are liable, and has established that marine chronometers obey with great regularity the combined action of the temperature, and the thickening of their oils, which arises from the lapse of time.

In order to comprehend the effect of this double influence, let us conceive a chronometer placed within an enclosed space, and maintained at a constant temperature: its daily rate will vary insensibly with the time, and if we designate by a, this rate at the commencement of the experiment, it will be $a + b\,x$, at the end of a number of days x, indicating by b the variation of the daily rate in one day.

* The greater part of the above remarks and information have been freely translated from M. De Cornulier's work, originally published, as already mentioned, in the "Annales Maritimes," and for the most part reprinted in the "Recherches Chronométriques," pp. 87–175. To his valuable work we are also greatly indebted for many useful extracts which enrich these pages.

† "Recherches sur les Variations de la Marche des Pendules et des Chronomètres: par M. Aristide Lieussou, Ingénieur Hydrographe de la Marine," &c. &c. "Extrait des Annales Hydrographiques (1853). Paris, 1854." Copious extracts from this work are also given in the "Recherches Chronométriques," pp. 216–263.

M. Lieussou attributes this variation proportional to the time, to the defect of isochronism of the balance-spring. Since, in general, chronometer-makers arrange their balance-springs so that the small oscillations should be more rapid than the great ones, we may say that the defect of isochronism determines in general, an acceleration in the daily rate of a chronometer. According to this explanation, the constant b, would give the measure of the precision with which the isochronism of the oscillations has been established. In the greater number of chronometers intended for sea service, this quantity rarely attains to one-hundredth of a second, and it appears to preserve the same value so long as the balance-spring is left in the same state.

If the chronometer was always submitted to the same temperature, we should have then its daily rate by means of the expression $a + b\,x$: but when the temperature varies, this expression becomes more complicated, as we shall presently see.

Let us suppose that the artist should have adjusted his chronometer at 0° and 30° (centigrade), that is to say, that he should have determined the positions of the compensating weights on the balance-wheel, so that the daily rate should be exactly the same at these two extreme temperatures. If we should place this chronometer in a box, in which we could make the temperature vary from degree to degree from 0° to 30°, we should experience immediately an increasing acceleration in the daily rate, whilst the temperature should be comprised between 0° and the mean temperature 15°; but at 15°, the daily rate would attain a maximum value, and if the temperature should continue to increase from 15° to 30°, the daily rate would go on continually diminishing, until it attained at 30° the same value it had at 0°. This diminution of rate would still manifest itself for temperatures below zero, and above 30°, and it would be so much the more considerable, as we swerved more and more from the mean temperature 15°.

M. Lieussou has determined this fact in discussing the observations of sixty chronometers, followed out at the observatory at Paris. He has, moreover, remarked, that for an equal variation of temperature more or less, reckoned as a point of departure from the temperature T, the arithmetic mean between the two extreme temperatures, for which the artist has adjusted his chronometer, the rate diminishes by equal quantities. He has

investigated by graphic constructions the law of this variation, and he has found that, t being the actual temperature to which the chronometer is exposed, it is proportional to the square of the difference of temperatures T and t.

Thus designating by a, the daily rate corresponding to the temperature T, we shall have the daily rate m at any temperature whatever t, by means of the expression,

$$m = a - c\,(\mathrm{T} - t)^2$$

c being the constant variation that the daily rate a undergoes, when the temperature to which the chronometer is subjected changes from T to T $\pm$ 1°.

This constant c varies for different chronometers, but it would appear to preserve the same value for each chronometer, whilst its balance remains in the same state. It represents the precision with which the artist has adjusted the compensation. Its value is generally below $0^{s}{\cdot}015$ in good chronometers, purchased after trial for the public service.*

We see from the preceding discussion, that the correction $c\,(\mathrm{T} - t)^2$ only depends on the half sum T, of the extreme temperatures selected by the artist for the adjustment of the compensation, and that it remains the same, whatever may be the temperatures, provided that their half sum be T. It is for this reason, that without troubling ourselves with the two extreme temperatures by which the experiments relative to the adjustment have really been made, we may say that the chronometer has been adjusted for this mean temperature T.

The preceding expression furnishes a very simple means of appreciating the influence that the choice, apparently arbitrary, of the mean temperature T, exercises upon the rate of a chronometer. Taking the case of two chronometers A and B adjusted with the same success, a fact indicated by the equality of the coefficient

* In M. Lieussou's researches, thermometers with the centigrade scale are referred to. If any other kind of thermometers were used, the numerical values of the coefficient c, of the chronometer, would of course be altered; the principle of the formulæ would remain the same.

If c, T, and t, refer to the centigrade scale, and c', T′, and t', to Fahrenheit's, c and c' become comparative, and can be obtained the one from the other, by the expression,

$$c\,(\mathrm{T} - t)^2 = c'\left((\mathrm{T}' - 32^\circ) - t'\right)^2$$

$c = 0^{s}{\cdot}015$ of the centigrade scale, corresponds to $c' = 0^{s}{\cdot}0046$ for Fahrenheit's.

c. For the chronometer A, supposed to be adjusted for 8° and 38° (centigrade), whose mean T is 23°, the influence of the temperature will be represented by $c\,(23° - t)^2$.

In like manner, for a chronometer regulated for 0° and 26°, we should have for this influence $c\,(13° - t)^2$.

We see, by simple inspection, that when these two watches are kept at a place whose mean annual temperature is 13° (55°·4 Fahr.), as for instance, in the chronometer-room of the observatory at Paris, they would be very differently affected by changes of temperature. The chronometer B adjusted to 13° would have variations of rate much more feeble than the other, since the change of temperature would take place round and about 13°, the mean temperature of the place: thus at Paris B would be judged superior, but if we placed the two chronometers in a room whose mean temperature was 23° (73°·4 Fahr.) the contrary would happen. A would be considered superior to B.

The combined influence of the thickening of the oils* and of the temperature upon the rate of a chronometer, can therefore be represented by two terms, the one proportional to the time, the other proportional to the square of the difference of temperature

* "The thickening (or gradual evaporation) of the oils (with which the pivots are lubricated), which takes place by lapse of time, by augmenting the resistance, whilst the impulse to which it ought to be in equilibrium remains constant, tends to diminish the amplitude of the vibrations of the balance. The mass of the balance being very small, and the amplitude of the vibrations very great, this effect is naturally considerable; thus the arc of vibration being at a mean, 415°, when the oils are fresh, it is no more than 330°, when the oils are aged three years. If this great alteration of vibration affects but little the rate of a chronometer, it is only because the vibrations of the balance-spring, which determine the alternate movements of the balance, have sensibly the same duration in the great arcs of 415°, and in the small arcs of 315°.

The employment of the balance-spring, as the regulator of the movements of the balance, is only then founded on a remarkable property which it enjoys. If it is short, the great vibrations are more rapid than the small ones. If it is long, the small are more rapid than the great. There is therefore in every balance-spring a length which gives to the great and small oscillations the same duration: for each chronometer this length is determined by trial, by comparing the rates corresponding to two extreme vibrations, which are obtained by varying the moving force or the tension of the main spring. In practice this special length is never rigorously obtained, and in other respects the thickening of the oils, which diminishes the amplitude of the oscillations, by 100°, in three years, alters their duration considerably." (Lieussou, p. 27.)

In M. Lieussou's system, the term, $b\,x$, in his "equation of rate," represents the accumulation of the acceleration due to the influence of time, and although chiefly viewed as a function of the age of the oils, it in reality represents the product of all the irregularities which supervene by time,—wear, dirt, friction, &c.

reckoned from the fixed temperature T, peculiar to each chronometer, and the daily rate may be calculated by aid of the expression,

$$m = a + b\,x - c\,(\mathrm{T} - t)^2,$$

in which, m represents the daily rate of the chronometer at the temperature t, after x days elapsed since the epoch, for which the daily rate was a for the temperature T, which has served for the adjustment of the compensation.

M. Lieussou has arrived at this equation by graphically constructing the curves of the daily rates of a great number of chronometers, followed up for a year at the Observatory at Paris: he has taken the intervals of time for abscissæ, and for ordinates the daily rates observed at each epoch.

For the purpose of disentangling the movement of the watches from the influence of the variations of temperature, the author considers in the curve of a chronometer, the points whose ordinates represent the daily rates observed at the same temperature, and he finds that these points are situated in a straight line; for another temperature, the points of the curve are also in a straight line, parallel to the first, so that by cutting the curve of the daily rates of the chronometer by a series of parallel straight lines, making with the line of the abscissæ an angle peculiar for each chronometer, the ordinates of the points of intersection, represent the daily rates, corresponding to the *same* temperature, the temperature varying when we pass from one straight line to another. M. Lieussou concludes from this, that the rate of a chronometer, submitted to a constant temperature, varies as the ordinate of a straight line, and may be expressed each day by the expression $a + b\,x$.

With respect to the influence of variations of temperature on the daily rates, it is represented in the curve of a chronometer, by the distances which separate the parallel straight lines of which we have spoken, these distances being measured upon the ordinates themselves. After some trials, M. Lieussou has recognised that these distances vary in the direct ratio of the square of the difference between the actual temperature, to which the chronometer has been exposed, and a certain temperature corresponding to the greatest observed daily gaining rate (*avance*): we have seen from the remarks which have preceded, that this

temperature occupies precisely the mean place between the extreme temperatures which have been employed in determining the position of the compensating weights on the balance.

The equation of the daily rate of a chronometer considered as a function of the time and temperature, involves therefore the four constants a, b, c, and T, of which we now know the signification. These constants can be determined for each chronometer (as we shall show further on) by means of four daily rates accurately observed at temperatures very different; and we could obtain their values more exactly by a greater number of precise observations, furnishing a series of equations of condition. M. Lieussou has done this for many chronometers presented for public trial at the Observatory. For the greater part of them, the daily rates calculated by the aid of the formula, agree throughout the whole period of the competition, almost within two or three tenths of a second, with the observed daily rates. Although under certain circumstances, the daily rate may have varied 15^s in a year, or even 12^s in three months, the differences between the calculation and the observation rarely exceeded half a second.*

The researches of the author have not hitherto been extended beyond observations on chronometers followed up for a year in an observatory; but for a watch which has been exposed to all the accidents of a long voyage, experience can alone decide (observes the report of the committee to whom the work was referred) whether the constants determined before departure would serve for the whole voyage, or whether it would not be necessary to ascertain their values afresh. Nevertheless it seems natural to suppose that the constants, T and c, which depend on the adjustment of the compensation, could be obtained in a few days, by means of experiments made at temperatures very different, and then two daily rates, deduced from observations made during the

* "Among the chronometers whose movements were considered by the author, he found many which, not having satisfied the conditions of the public trial, had been rejected for purchase by the naval department. Notwithstanding this, the method of calculation had been applied to them with equal success. The influences of time and temperature had caused in their rates considerable variations, but these variations had taken place in a manner almost as regular as in the best instruments. Thus making allowance for the particular irregularities that a watch would undergo at sea, these chronometers would have given by means of the empirical equation, longitudes as exact as the others." (Lieussou, p. 17.)

voyage under favourable circumstances, and at intervals of time sufficiently great, would suffice to give the value of the constants a and b, which belong to each chronometer embarked. This calculation which is sufficiently simple, (as we shall show further on,) could be made by the officer charged with the care of the watches.

Determination of the Constants which enter into the Equation of the Rate of a Chronometer.

The general equation of the rate of a chronometer as a function of the time and temperature, considered as independent variables, is

$$m^s = a^s + b^s x - c^s (T^\circ - t^\circ)^2$$

in which formula m expresses the actual daily rate at any moment, at a given temperature t°.

a is the initial daily rate of the chronometer at the temperature T°. It is the measure of the imperfection of the adjustment of the watch to mean time, at the temperature T.*

The constant T is the special temperature at which the chronometer takes its maximum rate, it is, as we have before stated, the arithmetic mean of the two temperatures for which the artist has established the equality of the rate; for a well-regulated chronometer this constant ought to be comprised between 15° and 20° (59° to 68° Fahrenheit).

The coefficient c is the diminution of the daily rate for a change of temperature of one degree centigrade, more or less, reckoning from T°. It is the measure of the imperfection of the compensation, and remains invariable, so long as the balance-spring and the balance are not modified; for a good chronometer this coefficient ought not to exceed $0^s{\cdot}02$.

* "This initial rate augments or diminishes by a quantity $b\,x$, proportional to the number of days elapsed x; b being generally *positive* (but not always), and less than $0^s{\cdot}01$. Artists are in the habit of adjusting the initial rate some seconds slow on mean time, in order that at the end of three years or so (for which period the oils are supposed to last) it may vary from mean time as little as possible. Assuming for instance that a equals $-5^s{\cdot}00$ when the chronometer leaves the hands of the artist, it will be equal to $-5^s{\cdot}00 + 548\ b = 0^s{\cdot}00$ nearly, at the end of eighteen months (or 548 days); and will be $-5^s{\cdot}00 + 1095\ b = +\ 5^s{\cdot}00$ nearly, after three years (or 1095 days); in cases where the coefficient of the acceleration, b, is positive, as is usually the case." (Lieussou, p. 101.)

The coefficient b is the change of rate of a chronometer in a unit of time. It would appear to vary a little after a long interval, but it is sensibly constant during a year. It may, perhaps, hence be considered, as the measure of the defect of isochronism between the great and small oscillations of the balance; for a good chronometer it ought not to exceed $\pm 0^s{\cdot}01$ per day, or $\pm 0^s{\cdot}30$ per month.

The four constants, a, b, c, and T, which enter into the general equation, and whose particular values for each chronometer constitute its "special characteristics" (*son régime spécial*), can be determined by any four rates whatever, observed at different temperatures and different epochs.

Let m_1, m_2, m_3, m_4 be the four observed rates, corresponding to the four temperatures t_1, t_2, t_3, and t_4, let us suppose, to facilitate the calculation of the constants, separated by equal intervals h.

We have then the four equations of condition,

$$\begin{aligned}
m_1 &= a + 0\,b - c\,(\mathrm{T} - t_1)^2 \\
m_2 &= a + h\,b - c\,(\mathrm{T} - t_2)^2 \\
m_3 &= a + 2\,h\,b - c\,(\mathrm{T} - t_3)^2 \\
m_4 &= a + 3\,h\,b - c\,(\mathrm{T} - t_4)^2
\end{aligned}$$

whence, adding together the first and third, and subtracting twice the second,

$$m_1 - 2\,m_2 + m_3 = - c\,\{t_1^2 - 2\,t_2^2 + t_3^2 - 2\,\mathrm{T}\,(t_1 - 2\,t_2 + t_3)\}$$

adding together the second and fourth, and subtracting twice the third,

$$m_2 - 2\,m_3 + m_4 = - c\,\{t_2^2 - 2\,t_3^2 + t_4^2 - 2\,\mathrm{T}\,(t_2 - 2\,t_3 + t_4)\}$$

adding together the third and fourth, and subtracting the first and second,

$$m_3 + m_4 - m_1 - m_2 = 4\,h\,b - c\,\{t_3^2 + t_4^2 - t_1^2 - t_2^2 - 2\,\mathrm{T}\,(t_3 + t_4 - t_1 - t_2)\}$$

adding together the four equations,

$$m_1 + m_2 + m_3 + m_4 = 4\,a + 6\,h\,b - c\,\{(\mathrm{T} - t_1)^2 + (\mathrm{T} - t_2)^2 + (\mathrm{T} - t_3)^2 + (\mathrm{T} - t_4)^2\}$$

which for shortness may be put under the form,

$$\alpha \;=-\,c\,(\beta - 2\,\mathrm{T}\,\gamma)$$
$$\alpha' \;=-\,c\,(\beta' - 2\,\mathrm{T}\,\gamma')$$
$$\alpha'' = 4\,h\,b - c\,(\beta'' - 2\,\mathrm{T}\,\gamma'')$$
$$\alpha''' = 4\,a\; + 6\,h\,b - c\,\{(\mathrm{T}-t_1)^2 + (\mathrm{T}-t_2)^2 + (\mathrm{T}-t_3)^2 + (\mathrm{T}-t_4)^2\}$$

From the two first of these equations, which only involve the two unknowns c and T, we obtain by reduction,

$$\mathrm{T}^{\circ} = \frac{1}{2}\,\frac{\alpha\,\beta' - \alpha'\,\beta}{\alpha\,\gamma' - \alpha'\,\gamma} \qquad (1)$$

$$c^{s} = \frac{\alpha\,\gamma' - \alpha'\,\gamma}{\beta'\,\gamma - \beta\,\gamma'} \qquad (2)$$

T and c being thus known, we have from the third equation,

$$b^{s} = \frac{1}{4h}\,\{\alpha'' + c\,(\beta'' - 2\,\mathrm{T}\,\gamma'')\} \qquad (3)$$

and from the fourth,

$$a^{s} = \frac{1}{4}\,\{\alpha''' - 6\,h\,b + c\,\{(\mathrm{T}-t_1)^2 + (\mathrm{T}-t_2)^2 + (\mathrm{T}-t_3)^2 + (\mathrm{T}-t_4)^2\} \qquad (4)$$

These four equations determine the four constants, as functions of the daily rates and temperatures observed at four epochs (separated for the convenience of calculation by equal intervals).*

* It will be instructive to give an illustration of the application of the formulæ.

The records of observations of the chronometer, No. 200 Winnerl, followed uninterruptedly at the Observatory at Paris, from June 1st, 1847, to Sept. 1st, 1848, supply the following data:—

Date.	Daily Rate. (Mean in Ten Days.)	Daily Temperature. (Centigrade.) (Mean in Ten Days.)	Equations of Condition.
	s	°	
25th Oct. 1847	+1·18	+15·0	$+1{\cdot}18 = a + \;\;0\,b - c\,(\mathrm{T}-15^{\circ})^2$
25th Jan. 1848	−1·19	+ 1·0	$-1{\cdot}19 = a + \;91\,b - c\,(\mathrm{T}-\;1)^2$
25th April, 1848	+1·53	+13·0	$+1{\cdot}53 = a + 182\,b - c\,(\mathrm{T}-13)^2$
25th July, 1848	+1·62	+21·0	$+1{\cdot}62 = a + 273\,b - c\,(\mathrm{T}-21)^2$

Treating the equations of condition as pointed out above, we shall obtain,

	$\alpha \;= +\,5{\cdot}09^{s}$		$\beta = +\,392$
	$\alpha' = -\,2{\cdot}63$		$\beta' = +\,104$
	$\alpha'' = +\,3{\cdot}16$		$\gamma = +\;\,26$
	$\alpha''' = +\,3{\cdot}14$		$\gamma' = -\;\;\,4$
Also	$\beta'' = +\,384$	and	$\gamma'' = +\,18$

M. Lieussou states, moreover, as the result of his experience, that for a chronometer placed in an observatory, the equation,

$$m = a + b\,x - c\,(T - t)^2$$

gives without sensible error, the mean rate, in any interval which does not exceed a month, when we put for t, the mean temperature observed in that interval.

Otherwise we ought to put for m_1, m_2, m_3, and m_4; t_1, t_2, t_3, and t_4; the rates and mean temperatures observed in ten days,

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Hence

$$T = \frac{1}{2}\,\frac{\alpha\,\beta' - \alpha'\,\beta}{\alpha\,\gamma' - \alpha'\,\gamma}$$

becomes

$$T = \frac{1}{2}\;\frac{529 + 1031}{-20{\cdot}36 + 68{\cdot}38} = \frac{1}{2}\;\frac{1560}{48} = 16^\circ{\cdot}25$$

and

$$c = \frac{\alpha\,\gamma' - \alpha'\,\gamma}{\beta'\,\gamma - \beta\,\gamma'}$$

becomes

$$c = \frac{48}{2704 + 1568} = \frac{48}{4272} = 0^{s}{\cdot}0112$$

Also

$$b = \frac{1}{4h}\,\{\alpha'' + c\,(\beta'' - 2\,T\,\gamma'')\}$$

becomes

$$b = \frac{1}{364}\,\{3{\cdot}16 + 0{\cdot}011\,(384 - 585)\}$$

$$= \frac{0{\cdot}949}{364} = 0^{s}{\cdot}0026$$

And

$$a = \frac{1}{4}\,\{\alpha''' - 6\,h\,b + c\,(\overline{T - t_1}^2 + \overline{T - t_2}^2 + \overline{T - t_3}^3 + \overline{T - t_4}^2)\}$$

$$= \frac{1}{4}\,\{3{\cdot}14 - 1{\cdot}42 + 0{\cdot}011\,(267{\cdot}25)\}$$

$$= \frac{1}{4}\,(4{\cdot}66) = 1^{s}{\cdot}16.$$

This value of a, "the initial rate," at the temperature T, corresponds to the epoch Oct. 25th, 1847.

For any other date it will be given by the formula,

$$m = a + b\,x$$

Thus for the 25th Jan. 1848 (91 days afterwards) it would be

$$\begin{aligned} m &= 1^{s}{\cdot}16 + 0^{s}{\cdot}0026 \times 91 \\ &= 1{\cdot}16 + 0{\cdot}2366 \\ &= 1{\cdot}3966 \\ &= 1{\cdot}40 \end{aligned}$$

The equation of this chronometer therefore, deduced from four mean rates of ten

twenty days, thirty days, for the purpose of withdrawing the constants from the inevitable errors of isolated observations. The interval which seems to embrace the greatest precision, would appear to be thirty days for a good chronometer, and twenty days for one moderately good.

M. Lieussou gives numerous tables showing the application of his formulæ to chronometers under trial at the Observatories at Paris and Greenwich, and certainly the accordance of the calculated and observed rates is very striking. As the result of his

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days' interval, separated by intervals of three months, and referred to the 25th Jan. 1848, would be,

$$m = 1^s{\cdot}40 + 0^s{\cdot}0026\,x - 0^s{\cdot}011\,(16^\circ{\cdot}25 - t)^2$$

M. Lieussou afterwards proceeds to investigate the equation of rate of this chronometer by other observations differently combined, and shows that within allowable limits of errors of observation, similar results are obtained. (Lieussou, p. 56, &c.)

The following table, selected from the numerous examples in M. Lieussou's work, will show the application of his system, and how closely it agrees with observation.

Comparative Table of Mean Daily Rates (in a month) observed and calculated.

$$a = +\,6^s{\cdot}50\,;\ b = 0^s{\cdot}005\,;\ c = 0^s{\cdot}045\,;\ T = 16^\circ{\cdot}5.$$

Epoch of Initial Rate April 15th.

Date.	Mean Temperature. (Centigrade.)	Chronometer No. 83, Winnerl. $m = 6^s{\cdot}50 + 0^s{\cdot}005\,x - 0^s{\cdot}045\,(16^\circ{\cdot}5 - t)^2$				
		$6^s{\cdot}50 + 0^s{\cdot}005\,x$	$-0^s{\cdot}045\,(16^\circ{\cdot}5 - t)^2$	Daily Rate. Calculated.	Daily Rate. Observed.	Daily Rate. Difference.
1844.	°	s	s	s	s	s
April	13·3	+ 6·50	− 0·46	+ 6·04	+ 6·01	+ 0·03
May	15·4	+ 6·65	− 0·05	+ 6·60	+ 6·95	−0·35
June	18·4	+ 6·80	− 0·16	+ 6·64	+ 6·62	+ 0·02
July	20·2	+ 6·95	− 0·62	+ 6·33	+ 6·39	−0·06
Aug.	19·3	+ 7·10	− 0·35	+ 6·75	+ 6·98	−0·23
Sept.	18·4	+ 7·25	− 0·16	+ 7·09	+ 7·34	−0·25
Oct.	15·3	+ 7·40	− 0·06	+ 7·34	+ 7·44	−0·10
Nov.	11·4	+ 7·55	− 1·17	+ 6·38	+ 6·61	−0·23
Dec.	4·6	+ 7·70	− 6·37	+ 1·33	+ 1·00	+ 0·33
1845. Jan.	5·4	+ 7·85	− 5·54	+ 2·31	+ 2·37	−0·06
Feb.	4·5	+ 8·00	− 6·48	+ 1·52	+ 0·24	+ 0·28

("Lieussou," p. 62.)

researches the author lays down, first, "that four rates and four temperatures observed at intervals of ten days determine the four constants which constitute the special characteristics of each chronometer with a precision sufficiently remarkable;" and, secondly, "that four mean monthly rates and temperatures determine them with a rigorous exactness."

Adverting to what was before stated on the probable permanence of the constants T and c, which depend on the system of compensation, and on the subsequent determination of the other two, a and b, by observations made by the officer charged with the care of the chronometers, it may be advisable to enter a little more into detail.

The constants T and c being known by experiments previously made by the maker, or at an observatory, before delivery of the instruments on board ship; two observations for the rate, observed under favourable circumstances of differences of temperature, and at a convenient interval, would establish the equations of condition,

$$m_1 = a + o\,b - c\,(\mathrm{T} - t_1)^2$$
$$m_2 = a + h\,b - c\,(\mathrm{T} - t_2)^2$$

Taking their difference,

$$m_2 - m_1 = h\,b - c\,\{(\mathrm{T} - t_2)^2 - (\mathrm{T} - t_1)^2\}$$

whence $$b = \frac{1}{h}\left\{m_2 - m_1 + c\,\{(\mathrm{T} - t_2)^2 - (\mathrm{T} - t_1)^2\}\right\} \quad (5)$$

Again taking their sum,

$$m_1 + m_2 = 2\,a + h\,b - c\,\{(\mathrm{T} - t_1)^2 + (\mathrm{T} - t_2)^2\}$$

whence $$a = \frac{1}{2}\left\{m_1 + m_2 - h\,b + c\,\{(\mathrm{T} - t_1)^2 + (\mathrm{T} - t_2)^2\}\right\} \quad (6)$$

From these two equations (5) and (6), made at any time during the voyage, the constants a and b become known, T and c having been previously determined.

The initial rate a thus found, is the rate at the temperature T, at the epoch corresponding to the first of the two observed daily rates, used in the calculation.

Application of the Equation of Rate of a Chronometer to the Measurement of Meridian Distances.

We will now proceed to consider the application of Lieussou's formula, giving the "equation of rate" of a chronometer, to the measurement of the meridian distance between two stations, and compare it with the common method, which assumes the invariability of the rate during the interval, or at any rate its variation proportional to the time elapsed.

Taking in the first instance the most simple case, and assuming that the variation of temperature having on the whole been nearly uniformly progressive, it may be taken to have altered regularly by a daily quantity, p.

Then t_0 representing the initial temperature at the commencement of the voyage, on the day of departure, or at the epoch of the last observation for rate; after the lapse of x days, it would be $t_0 + p\,x$.

Substituting this in the general equation,

$$m = a + b\,x - c\,(\mathrm{T} - t)^2$$

we have,

$$\begin{aligned} m &= a + b\,x - c\,(\mathrm{T} - \overline{t_0 + p\,x})^2 \\ &= a + b\,x - c\,(\overline{\mathrm{T} - t_0} - p\,x)^2 \\ &= a + b\,x - c\,(\mathrm{T} - t_0)^2 - c\,p^2\,x^2 + 2\,c\,p\,x\,(\mathrm{T} - t_0) \\ &= a - c\,(\mathrm{T} - t_0)^2 + x\,\{b + 2\,c\,p\,(\mathrm{T} - t_0)\} - c\,p^2\,x^2 \end{aligned}$$

but the expression, $a - c\,(\mathrm{T} - t_0)^2$, represents the initial rate m_0, at the temperature t_0.

Hence $$m = m_0 + x\,\{b + 2\,c\,p\,(\mathrm{T} - t_0)\} - c\,p^2\,x^2$$

Deducing from this by integration, the expression for the error E at the date x, (the error at the commencement of the voyage being denoted by E_0).

$$\begin{aligned} \mathrm{E} &= \mathrm{E}_0 + \int m\,.\,d\,x \\ &= \mathrm{E}_0 + m_0\,x + \frac{x^2}{2}\,(b + 2\,c\,p\,(\mathrm{T} - t_0)\} - c\,\frac{p^2\,x^3}{3} \\ &= \mathrm{E}_0 + m_0 x + x^2\left\{\frac{b}{2} + \frac{c\,p}{3}\,(3\,\overline{\mathrm{T} - t_0} - p\,x)\right\} \qquad (7) \end{aligned}$$

The error E′ estimated on the hypothesis that the initial rate m_0 has remained constant during the traverse, would be,

$$E' = E_0 + m_0 x$$

and the error therefore of this estimated state of the chronometer, after a passage of x days, would be,

$$E' - E = - x^2 \left\{ \frac{b}{2} + \frac{cp}{3} (3\,\overline{T - t_0} - p\,x) \right\} \tag{8}$$

In order that this error should be nothing, it would be requisite that the coefficients b and c of the variation of the rate of the chronometer, with the time and temperature should be so also, which is never the case, at least with the coefficient c.

The preceding formula (7) would be conveniently applicable in practice, and would present no difficulty in numerical solution, in cases where the alteration of temperature having been nearly uniform, it might be assumed that it had varied from day to day by a given quantity, p.

Otherwise, if the temperature had been irregular, the daily rate might be determined for each day successively from the general equation,

$$m = a + b\,x - c\,(T - t)^2$$

or more conveniently substituting for a, its equivalent expression

$$m_0 + c\,(T - t_0)^2,$$

from the equation,

$$m = m_0 + c\,(T - t_0)^2 + b\,x - c\,(T - t)^2 \tag{9}$$

putting for t, the daily observed temperatures, t_1, t_2, t_3, &c.

The sum of these computed daily rates would then be the correction, to the error of the chronometer on local mean time, at the first station, to bring it up to the time of the second observation, to be employed in the deduction of the meridian distance between the two stations.

Without, however, proceeding so elaborately as this, the correction may be obtained in a manner more simple, and nearly as exact, by adding to the initial error E_0, n times the rate corresponding to the mean temperature $\frac{1}{n}(t_1 + t_2 + t_3 + \ldots + t_n)$; or even better still, by dividing the passage into convenient

intervals, five, six, or seven days for example, for each of which the rate corresponding to the mean temperature being determined, their mean is to be then finally employed for the computation of the accumulated rate, as the correction to E_0.

In dividing the whole period of the voyage into convenient intervals, the computer would be guided by circumstances, and would make them more or less long, according as the changes of temperature had been more or less abrupt. Taking then for each of these intervals, the rate due to its mean temperature as its mean rate, we should obtain without sensible error the state of the chronometer at the end of the traverse.

Let us take for example a chronometer whose equation of rate,

$$m = a + b\,x - c\,(\mathrm{T} - t)^2$$

is

$$m = a + 0^{\mathrm{s}}{\cdot}005\,x - 0^{\mathrm{s}}{\cdot}015\,(24^\circ - t^\circ)^2$$

and suppose a sea-passage of two months, during which the temperature has varied gradually from 0° to 21° (centigrade), or by 0°·35 per day on the average ($=p$).

Then its error at the end of the passage is given immediately by formula (7),

$$\begin{aligned}\mathrm{E} &= \mathrm{E}_0 - m_0\,x + x^2\left\{\frac{b}{2} + \frac{cp}{3}\left(3\,\overline{\mathrm{T} - t_0} - p\,x\right)\right\}\\ &= \mathrm{E}_0 + 60\,m_0 + 3600\,\{0{\cdot}0025 + 0{\cdot}005 \times 0{\cdot}35\,(72 - 21)\}\\ &= \mathrm{E}_0 + 60\,m_0 + 330^{\mathrm{s}}{\cdot}1\end{aligned}$$

If the duration of the traverse had been divided into six intervals of 10 days each, the six corresponding mean temperatures (in accordance with the supposition made above) would have been,

$$+1^\circ{\cdot}75\,;\quad 5^\circ{\cdot}25\,;\quad 8^\circ{\cdot}75\,;\quad 12^\circ{\cdot}25\,;\quad 15^\circ{\cdot}75\,;\quad 19^\circ{\cdot}25\,;$$

putting these successive values of the mean temperature, for t in formula (9),

$$m = m_0 + c\,(\mathrm{T} - t_0)^2 + b\,x - c\,(\mathrm{T} - t)^2$$

and substituting the above values of b and c, we have,

$$m = m_0 + c\,\mathrm{T}^2{*} + 0{\cdot}005\,x - 0{\cdot}015\,(24 - t)^2$$

* Since in this case $t_0 = 0$; $c\,(\mathrm{T} - t_0)^2 = c\,\mathrm{T}^2$.

Whence we obtain,

$$\left.\begin{array}{l} m_1 = \\ m_2 = \\ m_3 = \\ m_4 = \\ m_5 = \\ m_6 = \end{array}\right\} m_0 + 8^s{\cdot}64 + 0{\cdot}005 \left\{\begin{array}{r} 5 \\ 15 \\ 25 \\ 35 \\ 45 \\ 55 \end{array}\right\}^* - 0{\cdot}015 \left\{\begin{array}{r} (22{\cdot}3)^2 \\ (18{\cdot}8)^2 \\ (15{\cdot}3)^2 \\ (11{\cdot}8)^2 \\ (8{\cdot}3)^2 \\ (4{\cdot}8)^2 \end{array}\right.$$

Hence

$$\frac{m_1 + m_2 + \cdot \cdot \cdot \cdot + m_6}{6} = m_0 + 8^s{\cdot}64 + 0^s{\cdot}15 - 3^s{\cdot}29)$$
$$= m_0 + 5^s{\cdot}50$$

which is the mean rate of the traverse.

Consequently

$$E' = E_0 + 60\, m_0 + 330^s{\cdot}0$$

which agrees with the former result obtained by integration.

In the method usually followed for obtaining the longitude of a ship at sea, during a voyage, it is assumed that the initial daily rate m_0, determined before the departure, remains constant and invariable during the passage.

Hence we see, that in the case of the chronometer before us, such a supposition would involve in a passage of sixty days, an error of 330^s, or $1^\circ\ 22'\ 30''$ of longitude.

Again, the theory of a uniform alteration of rate in proportion to the time elapsed, as proposed by Borda and adopted by Tiarks† and modern navigators, leads to the employment of the mean of the rates, determined before departure and after arrival, in the measurement of the meridian distance, between the two terminal stations of the voyage.

Now in the case before us the initial rate is m_0; and the rate at the termination of the voyage, computed by the formula,

$$m = m_0 + c\,(T - t_0)^2 + b\,x - c\,(T - t)^2 \ddagger$$

will be found to be $m_0 + 8^s{\cdot}805$.

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* The whole duration of the voyage, 60 days, being divided into intervals of 10 days, the successive values of x corresponding to the middle of each period are 5, 15, 25, &c.

† See remarks on this subject further on, p. 153.

‡ $t_0 = 0$; $t = 21^\circ$, and $x = 60$ by supposition.

Hence the mean rate for the passage,

$$\frac{m_0 + (m_0 + 8^s{\cdot}805)}{2} = m_0 + 4^s{\cdot}402$$

and employing this to obtain the error of the watch,

$$E' = E_0 + 60\, m_0 + 264^s{\cdot}1.$$

The difference between this and the error estimated by Lieussou's formula is therefore 66^s or $16\frac{1}{2}'$ of longitude, which therefore represents in this particular case, the error arising from the assumption of a rate uniformly accelerated, compared with the calculation of the correction by his formulæ; premising of course that the terminal rate obtained by observation, really agreed with its computed value as above, $m_0 + 8^s{\cdot}805$, as Lieussou's experience has justified him in asserting that it would do, very nearly.

M. Lieussou has given in his work numerous other examples of the application of his formulæ, showing the near accordance of the results, whether obtained by the method of integration or by dividing the duration of the passage into convenient intervals, and deducing for them the corresponding series of mean rates. These examples, moreover, fully illustrate the serious errors that are involved in the usual assumption of the permanence of the initial rate, for longitudes at sea, or in the employment of the mean rate between that of arrival and departure, for the determination of the meridian distance between the terminal stations of the voyage. For the details of the calculations we must refer our readers to the work itself, which will be found also, in all its parts, well worthy of the attention of the scientific student.

The researches of this author also suggest some instructive facts for the consideration both of the artists who construct chronometers, and of the officers who are destined to employ them. The correction $c\,(T - t)^2$ depending on the system of the compensation, remains small, so long as the chronometer is maintained at a temperature nearly equal to T; as the actual temperature t varies from T, the correction increases, and the more rapidly the greater is its amount, in an increasing ratio. Hence, in the adjustment of a chronometer, the artist should consider the probable conditions of service or climate in which his instrument is likely to be employed, and make his arrangements for the adjustment of the compensation accordingly. Those also who may have to select chronometers for service at sea, and

especially when the equipment of a scientific voyage is under consideration, should bear this fact in mind, and choose chronometers for the voyage, whose values of T nearly correspond with the mean temperatures of the latitudes, in which the ship is destined to cruise. If these considerations are disregarded it may happen that a chronometer which would have been judged excellent in the tropics would be deemed detestable in high latitudes, or the reverse. The complaints, often made by commanders of ships, of the performance of their chronometers, have frequently, as Lieussou observes, no other foundation. The want of accord between the mean temperature to which they are subjected, and the value of T, selected for them by the artist, is their misfortune, and not their fault. It is clear also, that in all cases, the maintenance of uniformity of temperature is an important element in securing stability of rate.

The quality of the oils, of which the term expressing the effects of the acceleration, seems to be a material function, is also most important, and well worthy of the most careful attention of horologists.*

In conclusion we shall observe, that the application of M. Lieussou's formulæ may be very materially facilitated by the preliminary formation of tabular corrections, giving for each chronometer the values of the terms, bx, and $c\ (T-t)^2$, for the arguments x and t, the days elapsed since the commencement of

* The following extract from the Report of the Astronomer Royal to the Admiralty, on the annual trial of chronometers at the Observatory at Greenwich in 1860, has an important bearing on the above remarks :—

"An examination of the rates of the chronometers leads me to the following conclusions :—

"(A.) The material workmanship of all the chronometers is very good, and amongst nearly all the chronometers there is very little difference indeed in this respect. In uniform circumstances of temperature every one of the chronometers would go almost as well as an astronomical clock.

"(B.) The great cause of failure is the want of compensation, or the too great compensation, for the effects of temperature.

"(C.) Another very serious cause of error is brought out clearly in this trial; namely, a fault in the oil, which is injured by heat. This is very different with the chronometers of different makers. For instance, the oil used by one chronometer-maker (named in the Report) is not at all injured by heat; while some of that used by another chronometer-maker (also named) is so bad that, after going through the same heating as those of the first-mentioned maker, the rates of the chronometers are changed (on returning to ordinary temperature) by 80 seconds per week.

"(D.) I believe that nearly all the irregularities from week to week, which generally would be interpreted as proving bad workmanship, are in reality due to the two causes (B.) and (C.)"

the voyage, and the actual temperature. The constants b, c, and T, having been determined by previous observations.

The numerical values of these corrections, taken from the tables, being then substituted in the formula (9),

$$m = m_0 + c\,(\mathrm{T} - t_0)^2 + b\,x - c\,(\mathrm{T} - t)^2$$

would give the actual daily rate m, corresponding to the mean daily temperature t, and might then be immediately utilised for the determination of the longitude, in the daily navigation of the ship.

Method of Mouchez.

Since the appearance of M. Lieussou's work in 1853, the subject of chronometric research has engaged the attention of a French naval officer, M. E. Mouchez, and his views materially differing as they do from those of Lieussou, merit separate consideration. In the present state of chronometric science any researches, based on observation and experience, are valuable contributions for the elucidation of truth. Time and experience, which can alone settle the question, have not yet given any definite sanction to any one theory of analytical investigation in preference to another. Attention has only recently been called, or rather we should say redirected, to the importance of temperature corrections, and in consequence, careful observations of the temperature of the chronometers in most voyages, even including those of a scientific character, have not hitherto been sufficiently accurate or extensive, to furnish data for the corroboration of the empirical formulæ, which have been suggested by analysis, as the expression of the law which controls the movements of these instruments; we proceed therefore to explain Mouchez's views.*

Mouchez's experience has not rendered him favourable to the employment of algebraic formulæ in dealing with the performances of chronometers and the variations of their rates. "The variations so frequent and so diverse of the daily rates of almost all chronometers *embarked* (on board ship)," observes the author, "having demonstrated to me the complete insufficiency of all the

* "Observations Chronométriques, faites pendant la Campagne de Circumnavigation de la Corvette la Capricieuse, par E. Mouchez. Paris, 1855." This work has been in great measure reprinted in the "Recherches Chronométriques," p. 264, &c.

formulæ proposed up to the present time, to correct them, I have confined my attention to the graphic constructions themselves; for the different empirical formulæ, proposed for the correction of rates, are only translations more or less feeble of these graphic curves, and consequently can only represent them in an imperfect manner."

After describing at some length his system of graphical projection, by which he sought to exhibit to the eye, the movements of the chronometers, under the combined influence of time and varying temperature (for the details of which we must refer our readers to the work itself*), the author proceeds to comment on the conclusions to which the study of his chronometric charts has led him.

The annual sheets prove that the acceleration is very variable in sign and magnitude; that the influence of temperature is sometimes regular, sometimes irregular, in fact they seem to establish an important law, which has not hitherto been noted—this is "the augmentation by time of the thermometric sensibility of the chronometers, as if the compensation lost little by little its efficacious power.† This fact is revealed in the graphic constructions, by the progressive deviations of the isothermal curves. This influence of time (he adds) "is sufficiently singular to merit confirmation, and it would be interesting to make some researches respecting it."

With reference to the influence of alterations of temperature

* "Recherches Chronométriques," pp. 265–8.

† On this subject, De Cornulier commenting on the "progressive weakening of the compensation," discernible in the critical examination of some chronometric observations that he quotes, remarks, "Time alone is then the true cause which modifies the action of the compensations, and it does it always in the same sense, so that we may henceforth regard it as a newly-acquired fact, that a compensation which errs by defect becomes more and more inexact, whilst that which errs in excess tends to rectify itself.

"This remark ought to make us much more indulgent towards the artists whom we accuse of being negligent in the adjustment of their compensations; it is very possible that they only deliver to us watches perfectly adjusted; but time soon deranges their combinations, so that in practice most watches are insufficiently compensated."

After discussing the probable causes of these changes, and arguing that they cannot proceed from any actual alteration of mechanical condition in the metallic apparatus of the balance; this writer adds, "it is much more natural to seek for the cause of the effects that we have observed, in a change in the state of the oils; their progressive reduction by evaporation, perfectly corresponds to the variations, slow and progressive also, of the coefficients of temperature. On this hypothesis, the

on the rates, M. Mouchez thinks that "we may without great error adhere to the hypothesis generally admitted hitherto, of variations of rate, simply proportional to variations of temperature. This hypothesis is gratuitous it is true, but it is nearly conformable to facts, and has the advantage, moreover, by its simplicity, of being very easy for employment in practice." As to the acceleration, the author is of opinion, that it is unnecessary to take a separate account of it; in short passages which do not exceed a month, it may be neglected on account of its smallness, and in determining differences of meridians, where we employ the rates both of arrival and departure, its effects are mixed up with, and involved in, those arising from variations of temperature.

The method* employed by Mouchez for the deduction of the meridian distance between two stations, differs but little in principle from that previously proposed by De Cornulier; in explaining it, therefore, we may conveniently adopt a notation nearly similar to that used by the latter, premising, however, that the matter is somewhat simplified by not separately taking cognizance of the acceleration. The coefficient of temperature y is obtained immediately from the observations, by a comparison of two daily rates obtained during the voyage at different temperatures; the rate observations being combined in pairs for this purpose, (including as great ranges of temperature as possible,) in the most advantageous manner, according to circumstances. Then the difference between the two rates, being divided by their

metallic apparatus after a time, is no longer adjusted so as to compensate for the total effects of the temperature on the balance and balance-spring, but only for that portion which remains, after a part of it has been destroyed by the effects in a contrary sense, produced by the oils. If the latter are reduced, little by little by evaporation, their compensating action diminishes with their volume, and the metallic apparatus, which works always in the same manner, becomes more and more insufficient." (Rech. Chron. p. 132.)

These views are in accordance with the theory advanced by Lieussou, and seem to satisfactorily explain the phenomena in question.

* The author, who does not use algebraic formulæ, thus explains his mode of proceeding, "I have compared the mean temperature of the sea-passage with that of the stay in port, and multiplying this difference by the coefficient of temperature of each of the chronometers, I have found the correction to be made to its rate at departure to have that of the traverse. Performing the same operation on the rate of arrival, I have obtained a second rate of the traverse, which was rarely identical with the first one, on account of accidental variations, and of the acceleration; but the mean of these two rates ought to be as near as possible to exactitude, and it is with this that I have determined the difference of meridians of the two points." (Rech. Chron. p. 269.)

corresponding difference of temperature, gives the value of the coefficient y, for one degree of the thermometric scale.

Let m be the daily rate at departure, at the temperature t_1 : $m + n$, that of arrival, at the temperature t_2 : t_3 the mean temperature of the traverse, and q the number of days elapsed.

Then employing the rate obtained before departure, we have for the correction of the error of the chronometer at the first station,

$$\text{Corr.} = q\,m + q\,(t_1 - t_3)\,y \qquad (1)$$

employing the rate found after arrival,

$$\text{Corr.} = q\,(m + n) + q\,(t_2 - t_3)\,y \qquad (2)$$

and employing the mean rate

$$\text{Corr.} = q\left(m + \frac{n}{2}\right) + q\left(\frac{\overline{t_1 - t_3} + \overline{t_2 - t_3}}{2}\right)y$$

$$= q\left\{\left(m + \frac{n}{2}\right) + \left(\frac{t_1 + t_2}{2} - t_3\right)y\right\} \qquad (3)$$

Formula (1) may be usefully employed for the daily determination of the longitude at sea; (t_3 in this case representing the mean daily temperature of the chronometer since the departure.)

Formula (3) is adapted to the determination of the meridian distance at the termination of the voyage.

The following example illustrates its application to some observations made during the voyage of the Coquille:—

		h	m	s
Error of the watch before departure at Anhatomorin		+ 2	32	0
„ „	after arrival at Paita	+ 4	39	15

$$m = -5^{s}\cdot55 \text{ at temp. } t_1 = 20^{\circ}\cdot2^{*}$$
$$m + n = -5\cdot30 \text{ at temp. } t_2 = 22\cdot5$$
$$t_3 = 13^{\circ}\cdot1\,; \quad q = 145 \text{ days}\,; \quad y = 0^{s}\cdot522$$

Hence by formula (3),

$$\text{Corr.} = q\left\{\left(m + \frac{n}{2}\right) + \left(\frac{t_1 + t_2}{2} - t_3\right)y\right\}$$
$$= 145\,\{-5\cdot43 + (21\cdot3 - 13\cdot1)\,0\cdot522\}$$
$$= 145 \times (-1\cdot13)$$
$$= -163^{s}\cdot8$$

* Centigrade.

whence we have,

		h	m	s
Error at Anhatomorin	+	2	32	0
Correction	—		2	44
	+	2	29	16
Error at Paita	+	4	39	15
Difference of longitude	+	2	9	59

The author claims for this method the advantage, that it is very certain to correct the rate for its *actual* acceleration, whilst by the employment of formulæ, we are obliged to make use of a *mean* acceleration, based upon a great interval of time, and which consequently may not correspond with the epoch with which we are then dealing.

Mouchez adds that if the records of the daily comparisons indicated any abrupt variation of the rate of the watch during the passage, he measured its amount and took account of it according to its sign. Again, if during some time, there had been in the isothermal curves, any considerable variation, indicated by many observations, he determined for that epoch, a *special* coefficient of temperature, by aid of which the *cotemporaneous* meridian distances were subsequently obtained.*

"All this," he continues, "is doubtless only an affair of feeling one's way (*tâtonnement*), of approximation, and of probability; but this method seems much more appropriate to the study of an instrument still so imperfect, and subjected to causes of perturbation so various and unknown, than those which, founded on the invariability, entirely imaginary, of certain quantities termed *constants*, or on *a mathematical regularity*, in the variations produced by vices of construction, wish to embrace in a general formula all the movements of chronometers during entire years, and to assimilate, so to speak, the rate of this instrument to the regular phenomena of nature. This manner of employing chronometers will thus be at once more exact and more instructive than

* These we conceive are dangerous and reprehensible practices; they open the door to empirical treatment, and cooking of the observations, which is fatal to their integrity, as we may thus be tempted to bend them to the support of any desired solution. Much better to accept the actual indications of the chronometers for what they are worth, and reject their results altogether if they swerve beyond certain allowable limits.

that which has recourse to formulæ, since there are numerous small variations entirely irregular, of which it is impossible to take account in a general mathematical expression. Such a formula based upon the whole of the observations, can only be capable of correcting the error proceeding from the single cause, as a function of which it is constructed; and hence accidental variations must entirely escape correction.

"This is without doubt the reason," continues this writer in conclusion, "why they have always hitherto rejected on board ship the employment of different methods, proposed even by distinguished hydrographers, but who have not sufficiently remarked the difference that there is between a new chronometer observed on shore, and a chronometer embarked for some years, and subjected to a thousand causes of derangement. Thus a single observation, even a little defective, is supposed with reason to give more exactly the actual error of a chronometer, than a formula, whatever it may be, which must support itself on observations made at distant epochs."*

The author subsequently proceeds to make some strictures on the formulæ proposed by Lieussou. He contends that the conclusions derived from experiments made in an observatory on shore, on chronometers new, or recently cleaned, and whose oils are fresh, kept in absolute repose in a mild temperature, nearly constant, or varying only by slow degrees, cannot be taken as finally decisive. The conditions, under which chronometers perform their functions at sea, are frequently entirely different: there they are often subjected to changes of temperature abrupt, and extending beyond the limits in which they had been previously tried; they experience frequent petty shocks and trepidations, while their oils become aged; hence it happens that in practice at sea, we rarely find chronometers maintaining rates so admirably regular, as are to be found in the registers kept in observatories, or by chronometer-makers, and which lend themselves so complacently to formulæ of correction, containing terms which are functions of the temperature. Had M. Lieussou's formulæ been applied to the performances of chronometers at sea, this writer thinks, that the agreements between results calculated and

* We shall take occasion to show further on to what extent we concur in this reasoning, and in what degree we think it carried too far. (See p. 147.)

observed would have been found much less accordant.* It may also happen from the different modes of construction and adjustment adopted by different artists, that each collection of chronometers by the same manufacturer may have a peculiar manner of action, and that that which is true for the chronometers of one maker may be quite false for those of another.

Mouchez also states as a conclusion of his experience, that when the variations of temperature are sufficiently decisive to give importance to the corrections, the two expressions, $c\ (T - t)^2$, and $c'\ (t - t')$, give in reality results much less discordant than might be at first supposed.

In conclusion, this writer states his belief that these two suppositions, like all others of the same kind that can be imagined, are more or less erroneous, and that they have an efficacy very different, according as the chronometer is by one artist or another. He thinks that the idea admitted hitherto of variations simply proportional to differences of temperature, would destroy a great part of the error: that this hypothesis is of such simplicity and of such easy application at sea, that a very great superiority in any other method should be admitted, before we are induced to abandon it; we ought therefore to continue to employ it, until Lieussou's theory has been established by numerous observations *made at sea*, and especially rendered of easy application; for

* No doubt it is highly important that Lieussou's formulæ, and indeed any other, that may be proposed for the correction of the rates of chronometers, should be carefully tested by their application to observations *made at sea*. In order that this should be done satisfactorily, the zealous co-operation of seamen themselves is absolutely necessary. The want of this verification hitherto is no fault of the hydrographers, who have analytically investigated this matter on shore. Lieussou thus complains, "We would have desired to verify the equation of rate of a chronometer, as a function of the age of the oils, and of the temperature, by means of observations *made at sea*; unfortunately they have neglected up to the present time, even in voyages having a scientific character, to note the successive daily temperature to which the chronometers have been submitted on board. To procure this indispensible datum, we are reduced to make an hypothesis, and to suppose for example, that the unknown temperature of the chronometer-case has constantly varied as the mean of the temperature of the air and sea at noon." (Lieussou, p. 95.) The reader will not fail to observe, that what we want in this matter, is the actual mean daily temperature to which the chronometers are subjected in their case, and which is generally very different from the ever-varying level of the external or surrounding air. This temperature is best obtained by means of a maximum and minimum thermometer placed within the chronometer-case; the mean of its indications being taken to represent the mean temperature. It is much to be hoped that in future, more attention will be paid on board ship to the acquisition of this important element.

above all things, and as an absolute condition, it is necessary before any new method can be admitted, in the practice of observations at sea, that it should be *simple and expeditious.*

Hartnup's Method.

The subject of "Rating Chronometers" with special regard to the variations caused by changes of temperature, and with a view to the improvement of the daily practice of navigation at sea, has for some years past been an object of constant attention at the Liverpool Observatory, established at that great commercial port, by the munificence of the Town Council.

In addition to the usual astronomical work which forms part of the regular business of all observatories, the rating and testing of chronometers was one of the main objects for which the Liverpool Observatory was established; the increase of the security of navigation, and the safer conduct of long voyages, being rightly considered as matters of vital importance.

The Observatory being amply provided with all necessary apparatus, for conveniently testing chronometers, at all ranges of temperature, Mr. Hartnup, its able director, has devoted much attention to the systematic trial of chronometers before they are sent to sea, to verify their system of compensation, and to impress on all who may be concerned in their performances, the obvious connexion of fluctuations of rate with changes of temperature, and the necessity henceforth of taking account of it.

As is well known, the practice ordinarily pursued, when chronometers are embarked for a voyage, is for the ship to be furnished with a memorandum giving the rate of the chronometers as determined at the place of deposit on shore. It is assumed that this rate may be depended on, and that it will remain constant during the voyage. The navigation of the ship is then conducted with implicit reliance on this supposition, until the discrepancy between the longitude of the ship given by the chronometer, and the known longitude of the stations arrived at on making a landfall, often reveals the existence of serious errors, and is sometimes the cause of fatal disasters. For these, the chronometer is often wrongly blamed, whereas in truth its system of compensation not being exactly perfect, or its having been subjected to the influence of a range of temperature, exterior to the limits within which it had

been adjusted, it had only obeyed with faithful regularity, the natural laws of temperature which control its movements, and from which it cannot escape.

Mr. Hartnup proposes in future to furnish the mariner with a table of daily rates and corresponding temperatures, previously determined by experiments, which are to be employed in the daily navigation of the ship in lieu of a uniform daily rate, irrespective of temperature, as has hitherto been generally customary. This system, although a novelty in this country, at the present day, is virtually the same as that formerly proposed by the celebrated horologist, Pierre Leroy,* and in modern times, it has also been successfully practised by French naval officers in some recent voyages.†

* In 1754 Pierre Leroy sent to the Academy of Sciences at Paris the "description of a new watch suitable for use at sea," in which occurs the following passage: "The third method of avoiding the error caused by variations of temperature, and to which Ipropose to confine myself, is to fix a thermometer in the box of our watch, and to place it successively in hot-air chambers (*étuves*), and in places very cold: comparing then its variations with the degrees of the thermometer, we should write upon that instrument by the side of each degree the losing or gaining rate of the watch for this degree: by the aid of this precaution, the thermometer would always indicate the variations of the watch: now, in this case an error known is no longer an error. It would suffice then that the officer on duty should note in the register the degree of the thermometer, when he wound up the watch." (Lieussou, p. 11.) This is, to all intents and purposes, the course of experiment pursued at the Liverpool Observatory, and the method recommended by Hartnup, for adoption henceforth at sea.

† "The previous determination of the coefficient of temperature," observes M. De Cornulier, "seems to us so important, that we would wish to see all marine observatories provided with hot-air chambers and refrigerating apparatuses, in which should be placed maximum and minimum thermometers, for the purpose of testing the chronometers before their embarcation. It is a grave omission not to submit to this trial, those which are destined for employment in a scientific voyage. The knowledge of this coefficient would be even of great assistance in ordinary navigation, since all the applications that we could make of it, in the calculations relative to errors and rates, would then become of extreme simplicity, as soon as it was known.

"If in a place whose temperature was a, we should have determined the daily rate of a watch m, we could form a table of the rates that it would take at different temperatures, in establishing that for the temperature $a + b$, it would have the rate $m + n = m + b\,y$. To calculate the longitude of the ship in the next sea-passage, we should then take from this table, for each day of observation, the daily rate which corresponded to the mean temperature of the preceding day, and we should add it to the accumulated rate that we had obtained the preceding day, or subtract it according as the rate was gaining or losing, to determine the total variation that the watch had undergone since the day when its error was last ascertained. It is thus that we have operated on board the Allier, and this method was crowned there with complete success." (Recherches Chronométriques, p. 104.)

The application of this system at sea would be extremely simple, and presents no points of difficulty. A maximum and minimum thermometer placed within the chronometer-case, would indicate daily, at the time of winding, the actual temperature experienced by the chronometer during the previous day. The tabulated record of observed daily rates according to temperature, would then give immediately by inspection, the rate for the preceding day; applying this according to its sign, to the sum of the accumulated daily rates (similarly obtained), up to the previous day, we should have the whole amount of the accumulated rate on the given day, for the days elapsed since the departure from port, to be applied as a correction to the primary error of the chronometer, on starting for the voyage.

If the voyage were long, it would be advisable to verify the rates of the chronometers from time to time, by observation, at any convenient opportunity that might offer during the voyage. No doubt the rates so obtained might frequently differ from the rates previously determined at *corresponding* temperatures before departure, owing to the gradual influence of the acceleration, but there is every reason to believe that the *differences* of the rates, observed at *corresponding* temperatures, would always remain the same, or nearly so, while the chronometer continued in good condition.* Disregarding for the moment small variations due to the efflux of time and the influence of the acceleration, the experiments at the Liverpool Observatory prove that notwithstanding the great change of rate produced by change of temperature, in nineteen cases out of twenty the original rates return the instant the temperature is restored. On an average of about one chronometer in twenty it is found, that the rate changes in the most capricious manner with every change of temperature; but an examination of the records shows that the sea-rates of such chronometers are equally uncertain. Mr. Hartnup has published some tables of the rates of chronometers observed under varying

* On this subject, De Cornulier remarks, when speaking of the performances of the chronometers on board the Allier, "In the greater part of the watches the rates observed at each stay in port vary in consequence of the accelerations; but the change, with reference to one degree of the thermometer, is always the same constant coefficient y. The tables of the rate, that it is necessary to arrange after each period of rating, will no longer contain the same numbers, but these numbers will have between themselves, a constant difference from one degree to another." (Rech. Chron. p. 104.)

circumstances of temperature at the Observatory. From these it appears that the average change in the daily rate caused by changing the temperature from 60° to 40° (Faht.) was $6^{s}\cdot97$, while the change produced by altering the temperature from 60° to 80° was only $2^{s}\cdot85$, the variations, it will be seen, being much greater in the low than the high temperatures. Commenting on these facts, Hartnup observes, "It will be necessary for me to explain, that the changes of rate shown in these tables, notwithstanding the magnitude of a large majority of them, were very consistent with the change of temperature; so much so, that after tabulating the amount of change due to temperature, the future rate of any of them might be predicted with very great certainty. It appears, moreover, from the records, that this holds good when the chronometers are at sea. We find for instance, that the average of the rates of chronometers while on the voyage between Liverpool and New York, agrees with the average of the shore-rates of the same chronometers, in a temperature of about sixty degrees; and the average of the rates of chronometers belonging to ships employed in the African trade, agrees with the average of the shore-rates of the same chronometers, in a temperature of about eighty degrees. Now all this shows that the rates of chronometers vary with the temperature, both at sea and on shore; and the navigator who does not make himself acquainted with the variation of rate due to temperature, peculiar to his own chronometers, must on the average make longer voyages, and expose his ship to greater danger than he would do, were he to make himself acquainted with these facts."*

The application of this system to the measurement of "meridian distances" would be precisely analogous in all its features to that already described in speaking of the daily determination of the longitude of the ship at sea. The accumulated rate of the chronometer would be the sum of the tabulated daily rates corresponding to the observed daily temperature instead of the mean daily rate multiplied by the number of days elapsed as usually employed.

The advantages of this system seem so obvious that it is very desirable that every publicity should be given to it, and that it should be fairly tried at sea, not only as applied to the daily

* Report on the Liverpool Observatory, 1853.

determination of the longitude in the course of the voyage, but also, with a scientific aim in view, in the accurate measurement of meridian distances.

In the latter case it would be necessary to pay particular attention to the frequent redetermination of the rates, noting carefully their connexion with the temperature, and to take great care that the tabulated forms of predicted daily rates, depending partly on the primary table, and partly on the new observations, were constructed with minute regard to accuracy.

For the full development of this system, the seaman must have the hearty and truthful co-operation of the chronometer-makers themselves, or the assistance of observatories or other public establishments, furnished with the necessary apparatus, for carefully testing the actual performances of chronometers under varying circumstances of temperature. At present, in so far as we are aware, Liverpool is the only port in this kingdom * where, thanks to the enlightened liberality of its municipality, a well-organised establishment for testing chronometers, and informing the mariner as to their probable actual performances at sea, at present exists. Let us hope that the day is not far distant, when similar establishments maintained in some way, at the public expense, may exist, as an ordinary part of the machinery of navigation and commerce, at all our great naval and commercial ports, and that the benefits of this system may be better appreciated and more widely diffused.

Tiarks' Method.

English navigators, as we have before observed, have usually been in the habit of disregarding the refinements of temperature corrections, although, at least in modern times, fully recognizing change of temperature as one of the chief causes of marked changes of rate.

In ordinary navigation at sea, the uniformity and invariability

* A small establishment with similar objects in view, was organised by the Board of Trade a few years back, in the port of London. We regret to add, that partly from want of appreciation of its utility, and partly from the existence of professional prejudices, it failed to command that degree of support which would have justified the Government in continuing to maintain it, it was therefore, after a time, given up. Let us hope that this reproach to the intelligence of the shipping interests of London may soon be wiped away.

of the initial rate of the chronometer, determined before starting on the voyage, are generally implicitly relied on during its continuance; Lunars, the Lead, and a good Look-out, being trusted to, to help the mariner out of any difficulties that the infidelities of his chronometer might lead him into.

In the measurement of "meridian distances," when scientific accuracy is aimed at, it has generally been assumed that if any change of rate has taken place it has varied uniformly and in proportion to the time. This idea, originally it would seem advanced by Borda, has been the foundation of all the methods employed by modern navigators, with few exceptions, for correcting the results obtained from chronometers for changes of rate. Tiarks slightly extended its application, and showed, by a simple algebraic treatment, how it might be employed to correct, in a uniform and systematic manner, not only the whole meridian distance between the terminal stations of the voyage, but also the partial differences of longitude between the intermediate stations, where the errors might alone have been determined, but not the rates.

In cases where no attempt is made to correct the results by the chronometers for the errors produced by variations of temperature, or when the circumstances of the voyage, such as the shortness of the run, or prevalence of a uniform temperature, render such delicacy of treatment unnecessary, recourse must be had to Tiarks' plan, or to some similar system of treatment. Hence it will be necessary to enter at some length into its practical exposition. As this, however, will involve a good deal of detail, and the necessity for illustrating the formulæ by examples, its amplification must needs be reserved for a separate chapter.

Summary Remarks on the preceding Methods of correcting Chronometric Observations for Changes of Rate.

In the preceding pages we have placed at some length before our readers the modes of treatment for correcting the determination of the longitude at sea, or the measurement of meridian distances, obtained chronometrically, for the errors arising from changes of rate, under the combined and varying influences of time and temperature, proposed by different authorities, De Cornulier, Lieussou, Mouchez, Hartnup, and Tiarks.

Independently of a natural reluctance to pass a definite judgment on the views and opinions of authors of so much greater experience, and with so much more knowledge of the subject than himself, the writer of these pages is also restrained by the consideration that, in a matter of this kind, resting to a certain extent on an empirical basis, experience is wanting to afford grounds for a satisfactory and final opinion. Time and experience can alone decide what system of treatment is most suited for adoption, what formulæ of correction will prove most feasible in practice, and how far the so-called "constants," determined by observation, really continue permanent during long voyages. Unless skilfully and carefully handled, it is possible that the adoption of algebraic formulæ, involving numerical coefficients of minute value, determined by observation, cannot be successfully undertaken without incurring the risk, through a fanciful empiricism, of falling into errors greater even than those which it is our object to correct.

Some few observations to assist the general judgment of our readers will, however, probably be expected of us, and will not be out of place.

I. Change of temperature being now universally recognized as the principal cause of marked changes of rate, careful daily records of the actual temperature experienced by the chronometers during the periods of rating, and throughout the passages, become henceforth absolutely necessary and indispensable adjuncts in all reports of meridian distances. Even if the observer should not himself have employed them in his calculations, they should be furnished for the use of the hydrographers, who may subsequently have occasion to subject their results to a critical examination.

II. Great care should be taken to use all accuracy in noting and recording these mean daily temperatures. Vague or erroneous records would be worse than none; the corrections sought for are always minute quantities, and careless observations might introduce greater errors than they seek to cure.

III. In comparing the methods proposed by the different writers whom we have quoted, we would observe that De Cornulier and Mouchez derive their modes of correction from the immediate *sea observations* themselves,—a very great advantage, since observers will thus be enabled to correct their own

measurements at the time. Mouchez's plan is more simple than De Cornulier's, by blending into one correction the consideration of the effects of variation of temperature and the acceleration. To this there does not seem to be any valid objection in theory, the admissibility of empirical treatment being conceded. Lieussou and Hartnup's plans require the aid of a careful series of observations for the determination of their constants, or tabulated rates respectively, made either at an observatory, or by the artist who has adjusted the chronometers. It is probable that these constants, and also the law of connexion which links the variation of rate to changes in the thermometric scale, may vary insensibly with the progress of time, and their reverification might then be difficult or impracticable on foreign stations at long distances from home.

Both these methods require to have their utility and facile adaptation to practice, confirmed by experience gained at sea.

IV. Lieussou's formulæ, based upon the analytical investigation of numerous careful observations, are singularly ingenious, and in the hands of a skilful computer seem well adapted to the measurement and correction of meridian distances. They are well worthy of the attention of scientific navigators, and we trust they will receive the consideration they merit.

We cannot say that we concur in Mouchez's animadversions on Lieussou's formulæ, or in his hostility to algebraic formulæ in general. Algebra, the golden key which unlocks the portals of mathematical truth, is often only the symbolical expression of the plainest common sense. Its employment often clearly explains that which is obscure when expressed verbally. Mouchez's own views, even, are best explained, we think, by a simple algebraic treatment, as we have shown (*ante*, p. 136). Again, we do not think Lieussou's formulæ deserve his strictures, as to their inapplicability to practice, from their terms depending on elements deduced from byegone experiments; for, as we understand that gentleman's writings, he has by no means proposed that algebraic formulæ, involving constants derived from prior and distant observations, should supersede the use of rates derived at the time from present ones. The part they play in the measurement of meridian distances, if we have rightly apprehended him, is only meant to be auxiliary and subordinate to that performed by the

data derived from the immediate observations of errors and rates, obtained at the time of the passage under discussion.

With reference to Mouchez's remarks on the two expressions for the change of rate, considered as a function of the temperature, $c\,(T - t)^2$, and $c'\,(t - t')$, we think the former, depending on the normal temperature T, has a greater probability of correctness, because we know as an independent fact, that the artist has the power of adjusting the rate between certain limits of temperature, and that as we swerve from T, the mean of the two selected temperatures of adjustment, the errors of the rate rapidly increase. On the other hand the term $c'\,(t - t')$, representing the variation of rate for a given difference of temperature, is not the same in all parts of the thermometric scale, as clearly shown by Hartnup's experiments before mentioned; consequently the variation of the rate is not always correctly given by that expression, although it may be true, or nearly so, round and about the normal temperature T.

V. Hartnup's method, simple, and involving none but the easiest calculations, seems admirably adapted for use by the mercantile marine, in the daily practice of navigation at sea. In the merchant service, from divers obvious causes, facilities very often do not exist, for frequently checking the rates of the chronometers by fresh observations. On the other hand, the frequent return of merchant ships to their port of departure, affords the opportunity for reverifying the system of "tabulated rates" of their chronometers. This plan also possesses the important advantage, that it is simple, expeditious, and short. With proper precautions to ensure accuracy, as previously suggested, it might probably also be made available for the accurate measurement of "meridian distances."

VI. Mouchez's plan not differing essentially in principle, although simplified in form and expression, from that proposed by De Cornulier, seems well adapted for general practice; and, when skilfully combined with the formulæ of Tiarks' method, is that which, *for the present*, we recommend for adoption by English navigators.

Possibly experience may by and by decide more in favour of Lieussou's or Hartnup's methods, but at present they require the confirmation of practical trials at sea.

Very many persons, we doubt not, while admitting the patent irregularities produced by changes of temperature, will object altogether to the use of formulæ of correction, not so much on account of the additional trouble that they devolve on the computer, as because they seem to a great extent fanciful and empirical.

We would remind such objectors, that if they disapprove of formulæ for correcting for temperature, they may have the matter pretty much in their own hands, if they will be at the trouble to adopt suitable and obvious precautions.

If care is taken to maintain the chronometers at a uniform and equable standard (using artificial means if necessary), and if the errors and rates of the chronometers are redetermined by frequent observations, and the measurement of meridian distances, broken into as short links as possible and executed as rapidly as possible, the necessity for correcting for temperature and acceleration will well-nigh vanish altogether. The elimination of error is better than its correction, while the application of Tiarks' formulæ will satisfy all the wants of practice.

CHAPTER VII.

ON THE CHRONOMETRIC DETERMINATION OF MERIDIAN DISTANCES—TIARKS' FORMULÆ—COMBINATION OF MOUCHEZ'S PLAN OF CORRECTION FOR TEMPERATURE, WITH TIARKS' METHOD—COMPARISON OF FLINDERS' AND TIARKS' METHODS—METHODS OF VINCENDON DUMOULIN, COUPVENT DESBOIS, AND CHARLES PLOIX.

In the preceding chapter we have placed before the student the methods proposed by various writers for correcting the immediate results obtained by chronometric observations, for changes of temperature, and the influence of the acceleration.

English navigators generally, have not been in the habit of attending to these refinements. When the chronometers are kept in a uniform temperature, and when the runs are short, the changes which are produced by variations of temperature, and the influence of the acceleration, are minute quantities, which become mixed up with and masked by the petty irregularities of the observation, and even to a certain extent allowed for, when the mean of the rates at departure and on arrival is employed in the measurement of the meridian distances.

We proceed, therefore, to explain the mode of proceeding under general circumstances, no special account being taken of the influence of the temperature or the acceleration.

In the correction of meridian distances for the alterations which may have taken place in the rates of the chronometers employed, we propose to adopt the same hypothesis as we have already assumed* in the combination of observations for the determination of the rates, and to consider that any variation, which the rates may be found to have undergone between the two epochs for which they have been determined, has taken place uniformly and in proportion to the time. Of course, this

* See *ante*, p. 82.

idea is to be accepted under the same conditions as we have already laid down when treating of the combination of observations for the deduction of rates, and is to be taken with the same reservations, as to its ultimate and approximate truth, as we noted on that occasion.

The idea of an equable variation of rate, in proportion to the time elapsed, has usually been viewed by computers under one of two aspects.

It has either been considered that the change of rate increased or decreased uniformly by a given quantity from day to day, in which case the accumulated correction could be represented by the sum of an arithmetic series, in which the first term equals the common difference; or that, supposing the increment or decrement of rate to flow on uniformly from moment to moment, its accumulation could be expressed by the area of a right-angled triangle, whose base represented the time elapsed, and altitude the observed change of rate. We propose to adopt the latter supposition, consistently with what we have already done in treating of the combination of observations for rate at any given place, and to consider that any change of rate which is found to have taken place during a voyage between two epochs of rating at its commencement and termination, may be dealt with on precisely similar principles to those which we have shown to be practically admissible in discussing rate observations at any particular place.

The first idea, that of an arithmetic series, is that adopted by Flinders, King, Owen, and others; Tiarks, Fitzroy, and Bayfield* follow the latter hypothesis, viz. the area of a right-angled triangle.

It will be instructive to compare the two methods, and not difficult to show that the first idea will give the correction slightly in excess of what it ought to be; but in order not to interrupt the thread of our present discussion, this matter had better be reserved for investigation, at the close of this chapter.

Let the mean daily rate of a chronometer corresponding to a particular epoch before starting on a voyage $= a$.

* See "Voyages of Adventure and Beagle," Appendix to vol. ii. pp. 320–330; Owen's "Table of Longitudes," Explanation, p. 4; Forster's "Voyage," vol. ii. Appendix, p. 226; "Nautical Magazine," April 1843, p. 220; 1854, p. 169.

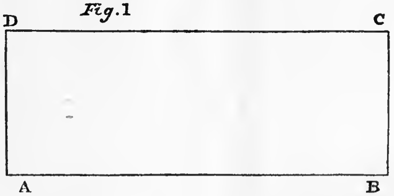
At the termination of the voyage, let the rate be again determined; the interval between the epochs to which the rates are referred being represented by t.

If it were found that the rate had not changed, but was still represented by a, the whole accumulation of the rate in the interval t would be given by the equation.

$$\text{Whole accumulation of rate} = t\,a.$$

Which again, as we have before observed, could be graphically represented by the area of a rectangle, whose base $AB = t$, the time elapsed; and altitude $BC = a$, the uniform rate.

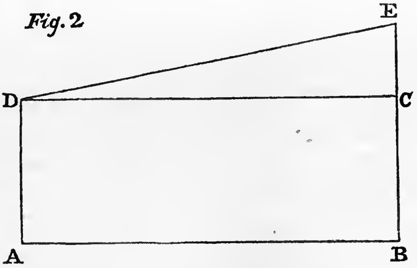
Fig. 1



$AB = t$
$BC = a$

If, on the contrary, it were found that the rate had changed to $a + b$,* on the assumption that the change from a to $a + b$ had taken place uniformly, and in proportion to the time, then, as we have formerly shown, the whole accumulation during the period t could be represented by the area of the figure ABED; that is, by the area of the rectangle ABCD added to the area of the triangle DCE.

Fig. 2



$AB = t$
$BC = a$
$CE = b$
$BE = a + b$

* a and b being either positive or negative, as the case may be: *gaining* rates, and the alteration of rates in a *gaining direction*, being considered *positive*; while *losing* rates, and the alteration of rates in a *losing direction*, are to be taken as *negative*.

Therefore, whole accumulation of rate during period t,

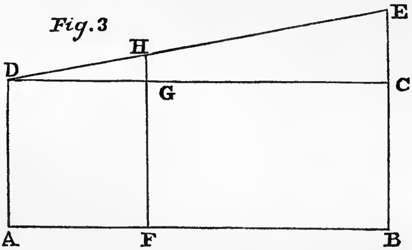
$$= t\,a + t\,\frac{b}{2}$$

$$= t\left(a + \frac{b}{2}\right)$$

and as the rate, $a + \frac{b}{2}$, is the mean of the former rate a, and the latter rate $a + b$, it at once follows, as Tiarks observes,* that the adoption of our hypothesis leads directly for the whole interval t, to the use of the mean of the rates at the commencement and termination of the voyage.†

In a similar manner, for any partial interval during the voyage τ, for which it is proposed to determine the correction, the whole accumulation of rate may be represented by the area of the figure A F H D, if A F $= \tau$, and if τ be supposed to commence from the epoch when a, the first rate, was determined; that is, at the commencement of the period t.

Fig. 3



A B $= t$
A F $= \tau$
B C $= a$
C E $= b$
B E $= a + b$

Now, the area A F H D is equal to the area of the rectangle A F G D, added to the area of the triangle D G H.

And by the principle of similar triangles, their areas being as the squares of their like sides,

$$\text{Area DGH} : \text{area DCE} :: \text{DG}^2 : \text{DC}^2$$

or

$$\text{Area DGH} : t\,\frac{b}{2} :: \tau^2 : t^2$$

$$\therefore \text{Area DGH} = \frac{\tau^2}{2\,t} \cdot b$$

Hence,

$$\text{Whole accumulation of rate for interval } \tau = \tau\,a + \frac{\tau^2}{2\,t}\,b$$

* Forster's "Voyage," vol. ii., Appendix, p. 226.

† This is also in accordance with the method formerly adopted by Fleurieu. "Voyage de Fleurieu," vol. ii. p. 541.

Suppose now that after starting on a voyage from a place A, where the rate a had been determined, the ship calls at two (or more) intermediate places, B and C, &c., and finally arrives at K, where a new rate, $a + b$, is obtained, and that observations for the errors on local mean time *only* were made at B and C (time or circumstances not permitting the rates to be determined anew at those places), it is required to investigate formulæ for the corrections due to the several meridian distances between the places A, B, C, &c., and K, using the observations for error at any two of them respectively, and the rates determined at the terminal points A and K.

Let the interval between the epochs for which the rates were determined at A and K $= t$, between the epoch at A and the observation at B $= \tau$, and between the epoch at A and the observation at C $= \tau'$, and so on.

Then, for the whole correction for the accumulated rate between the places A and K, where the rates were determined, we have the expression,—

$$\text{Correction} = t\,a + t\,\frac{b}{2} \qquad (\alpha)$$

For that between A and B,—

$$\text{Correction} = \tau\,a + \frac{\tau^2}{2\,t} \cdot b \qquad (\beta)$$

Similarly between A and C,—

$$\text{Correction} = \tau' a + \frac{\tau'^2}{2\,t} \cdot b \qquad (\gamma)$$

By taking the difference between (γ) and (β) we obtain that between B and C,—

$$\begin{aligned} \text{Correction} &= (\tau' - \tau)\,a + \frac{\tau'^2 - \tau^2}{2\,t} \cdot b \\ &= (\tau' - \tau)\,a + \frac{(\tau' + \tau)\,.\,(\tau' - \tau)}{2\,t} \cdot b \qquad (\delta) \end{aligned}$$

and similarly that from C to K,—

$$= (t - \tau')\,a + \frac{\overline{t + \tau'}\,.\,\overline{t - \tau'}}{2\,t} \cdot b \qquad (\varepsilon)$$

These several formulæ* are sufficient for all the cases which can arise in practice in the computation of meridian distances, where the rates have been determined at both ends of the chain. They are analogous to those employed by Tiarks,† and, we think, rather more simple in expression, and more satisfactory: since, by our mode of proceeding, the errors as well as the rates being referred to the same mean epoch, the notation employed admits of some simplification, and the formulæ can be expressed with more conciseness.

From what has been said above it follows, that if the rate of a chronometer has been found at the first terminal station only, the meridian distances between it and the places subsequently called at can only be approximately determined in the first instance, and will be liable to future correction should it afterwards appear that the rate of the chronometer has changed during the voyage.

But if the difference of longitude between any two stations be accurately known, and the rate at one of them only has been determined, then this known difference of longitude may be used as an element in the calculation, and be itself made subservient to the correction of the longitudes of places intermediately touched at between them; for, as we shall presently show, we then have the means of ascertaining the variation of the rate b, which may have taken place during the voyage, and hence can apply our formulæ (β), (γ), (δ), and (ε), to the cases in question.

This position may be made clear by the following simple algebraic treatment:—

Let M be the "difference of longitude," or the "meridian distance," between any two stations A and K, M being considered

* The number of places intermediately touched at on the voyage may be indefinite, care being taken that in the application of the formulæ generally A and K represent the points of arrival and departure where the rates are determined, and B, C, &c., any intermediate places whatever between them, where the errors on time *only* have been ascertained.

† See Forster's "Voyage," vol. ii., Appendix, p. 228. It will be observed on comparing the above with Tiarks' formula, that the expression $\frac{\tau^2}{2t} \cdot b$ in formula (β), while it is identical in form with his formula (2), is at the same time virtually the same as his more accurate expression given in formula (1), viz. $\frac{(\tau + d)\,\tau}{2t + d + d'} \cdot b$, in consequence of our adoption of the idea of the variation of the rate commencing at the mean epoch (that is, at the middle of the interval for which the rate is determined), and consequent alteration of notation.

positive or *negative*, according as the position of the place arrived at is *west* or *east* of that sailed from; and let λ and λ' be the errors of a chronometer on local mean time at the two places, the errors being reckoned as *positive* when considered *fast*, and *negative* when *slow*.

Then, if the chronometer kept true mean time,

$$M = \lambda' - \lambda.$$

But if the rate of the chronometer at starting from A were a,

Then $$M = \lambda' - (\lambda + t\,a)$$

which would be a true expression for M, if a remained constant, and an approximate one, if it were found on arrival at K that the rate had changed during the voyage, and become $a + b$; and in this case, in accordance with our hypothesis,—

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\}$$

Now in this expression, if M be assumed to be known, and b, the alteration of the rate, be taken as unknown; then, by transposition, we can find b from it.

For $$t\left(a + \frac{b}{2}\right) = (\lambda' - \lambda) - M$$

and $$a + \frac{b}{2} = \frac{(\lambda' - \lambda) - M}{t} \qquad (\zeta)$$

$a + \frac{b}{2}$, the *mean sea rate* for the interval, being thus known, and a having previously been determined by observation, b can easily be found,

Since $$b = 2\left(\overline{a + \frac{b}{2}} - a\right)$$

The whole change of rate b being thus known, it may be used by substitution in formulæ (β), (γ), (δ), and (ε)* to determine the respective meridian distances of the places previously touched at on the voyage.

* See *ante*, p. 154.

Again, if the error alone of the chronometer had been accurately ascertained at the commencement of the voyage (the pressure of business at the moment of departure, uncertain weather, or other adverse circumstances, preventing the corresponding determination of the rate), but the rate $a + b$, had been found at its termination; then, since $a + \frac{b}{2}$, the *mean sea rate*, becomes known from formula (ζ), a, the original rate at starting, and b, its alteration during the voyage, may readily be found.

Since $$a = 2\left(a + \frac{b}{2}\right) - (a + b)$$

and $$b = 2\left(\overline{a + b} - \overline{a + \frac{b}{2}}\right)$$

a and b being thus known, they can be applied as needful in formulæ (β), (γ), (δ), and (ϵ), as before.

Recapitulating our notation and formulæ for the sake of clearness and facility of reference, we have,—

Places called at during the voyage, being respectively represented by A, B, C, &c., and K.

$$\begin{aligned} \text{Rate at A} &= a \\ \text{K} &= a + b \end{aligned}$$

(a and b being positive or negative respectively, as the case may be).

$$\begin{aligned} \text{Error of chronometer at mean epoch at A} &= \lambda \\ \text{— \quad — \quad K} &= \lambda' \\ \text{at moment of observation at B} &= \lambda_1 \\ \text{— \quad — \quad C} &= \lambda_2 \end{aligned}$$

$$\begin{aligned} \text{Meridian distance A to K} &= M \\ \text{— \quad A to B} &= M_1 \\ \text{— \quad B to C} &= M_2 \\ \text{— \quad C to K} &= M_3 \end{aligned}$$

Interval between the epochs of the observations,

$$\begin{aligned} \text{A to K} &= t \\ \text{A to B} &= \tau \\ \text{A to C} &= \tau' \end{aligned}$$

Then, for the meridian distance from A to K, we have,—

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\} \tag{1}$$

from A to B,

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau a + \frac{\tau^2}{2t} \cdot b\right)\right\} \tag{2}$$

from B to C,

$$M_2 = \lambda_2 - \left\{\lambda_1 + \left(\overline{\tau' - \tau}\, a + \frac{\overline{\tau' + \tau} \,.\, \overline{\tau' - \tau}}{2t} \cdot b\right)\right\} \tag{3}$$

from C to K,

$$M_3 = \lambda' - \left\{\lambda_2 + \left(\overline{t - \tau'}\, a + \frac{\overline{t + \tau'} \,.\, \overline{t - t'}}{2t} \cdot b\right)\right\} \tag{4}$$

If the places intermediately called at during the voyage are more numerous than two, viz. B and C, as we have here supposed in the arrangement of these formulæ, their connexion with the points of departure and arrival, as well as with one another, can readily be obtained from the preceding formulæ by assigning their appropriate values to τ and τ', for the several places whose differences of meridians are sought.

Again, if the difference of longitude between the points of arrival and departure be assumed to be accurately known, then the *mean sea rate*,

$$a + \frac{b}{2} = \frac{(\lambda' - \lambda) - M}{t} \tag{5}$$

By the aid of which expression, if the first rate, a, at the place of departure, has alone been determined, b can be found.

For $$b = 2\left(\overline{a + \frac{b}{2}} - a\right) \tag{6}$$

And if a have not been determined, but $(a + b)$, the rate at the place of arrival has been found.

Then $$a = 2\left(a + \frac{b}{2}\right) - (a + b) \tag{7}$$

and $$b = 2\left(\overline{a + b} - \overline{a + \frac{b}{2}}\right) \tag{8}$$

The respective values of a and b being thus known, and employed by substitution in formulæ (2), (3), and (4), the meridian distances of places intermediately touched at during the voyage can readily be determined.

Observation I. Although in the application of the preceding formulæ, for the deduction of meridian distances, to cases in

actual practice, attention to the algebraic signs of a and b will of course always be necessary to ensure true results; yet, perhaps, the best course for computers, who may not have been accustomed to work from formulæ, to pursue, will be to take a plain practical view of the matter, and to apply the several corrections with reference to their obvious effects, rather than hamper themselves in their treatment by too close an attention to the apparent effects of the algebraic signs. Of course, when duly interpreted, the results by the two methods will always agree. Thus, if the error of a chronometer be *fast*, and the rate *gaining*, or *slow*, and the rate *losing*, the error will be increased by the effects of the accumulated rate, and the contrary if *vice versâ*. And again, if the rate, whether gaining or losing, is *increasing*, it must be remembered that the effects on the error of the two parts of the correction depending on a and b, will be in the *same* direction, and they must therefore be *similarly* applied; but if the rate, whether gaining or losing, is *decreasing*, then the contrary is the case, and the two parts of the correction must be applied *differently*.

Observation II. The intervals, t, τ, and τ', will usually be either integral numbers or simple decimal quantities, obtained by taking the differences between the dates of the epochs to which the observations are referred. But when the arc of longitude traversed during the voyage is considerable, it will be proper, when extreme accuracy is aimed at, to correct the intervals t, τ, and τ', by the fractions of a day, corresponding to the approximate difference of longitude, by means of the table given in the Appendix for converting intervals of time into decimals of a day.

For example, suppose the epoch of rating at the commencement of a voyage at Portsmouth is June $1^d{\cdot}875$ (the observations having been made at 9 A.M. mean time, on June 2d), and that on the termination of the voyage to the West Indies the rates were again obtained at Barbadoes, at the epoch July $6^d{\cdot}875$, *i.e.* at 9 A.M. on July 7th. Here the apparent interval or value of $t=35^d{\cdot}000$; but since Barbadoes is $3^h\ 45^m$ west of Portsmouth, the apparent interval, t, should in strictness be corrected by $0^d{\cdot}162$, the fraction of a day to which $3^h\ 54^m$ corresponds, and the true value of t to be substituted in the calculation will become $35^d{\cdot}162$, the interval being *increased* because the ship loses time by sailing to the *westward*.

In a similar manner, if the place of arrival is to the eastward of the point of departure, the apparent value of t should be *decreased* by the amount of the correction due to the approximate difference of longitude, because the ship gains time by sailing to the *eastward.*

Of course the apparent values of τ and τ' should, under like circumstances, be increased or decreased in a similar manner.

Again, if any of the errors or rates have been obtained by the method of "equal altitudes," the epochs of which refer to the moment of apparent noon or midnight at the place of observation, the intervals of time t, τ, and τ', obtained from them by comparison with other epochs, should in strictness be corrected for the variation in the "equation of time" in the interval; and the best mode of doing this in practice will doubtless be by reducing the moments of apparent time to the corresponding moments of mean time, by applying the equation of time for the given day to each day's observation separately, and then taking their differences as before, which will give the apparent values of t, τ, and τ', in mean solar time.

If the errors of the chronometers on time at the place have been obtained by transits or altitudes of the stars, it will be advisable to reduce the epochs to which they refer to their equivalent expressions in mean solar time, and then proceed as before.

Observation III. It will be observed, that where more than one chronometer is employed in the measurement of a meridian distance, the factors which multiply, a and b, in the formulæ, are common to all the chronometers,—a fact of considerable importance in the reduction of labour, and rendering far less formidable than appears at first sight the manipulation of a large number of chronometers in investigations of this nature. In fact, if it be thought desirable, the ultimate *mean* value of the several determinations of each chronometer taken separately may be obtained at once, by substituting in our formulæ, in lieu of the simple values of a and b for each chronometer, their mean value for the whole (regard being paid to their algebraic signs), and then proceeding as for a single chronometer, using also the corresponding mean values of λ, λ', λ_1, &c. The result will be the same as if one *fictitious* chronometer had alone been used, whose errors and rates represented the mean values of those appertaining to the whole number of chronometers actually employed.

This mode of proceeding is obviously less laborious than applying the corrections for, and obtaining the results of, each chronometer's performance taken separately. It has, however, the disadvantage of masking the results by, or the effects of the irregularity of, any particular chronometer in the general mean, and, therefore, of diminishing the ability of the computer to judge and discriminate between the separate performances of individual chronometers; at the same time, as affording a check on the accuracy of calculation, and as facilitating the computation of results in duplicate, the method has its value, and is worthy of being pointed out for the benefit of seamen.

Observation IV. Since the value of chronometric differences of longitude is entirely dependent on the assumed regularity of performance of the watches employed in the measurements, it is inexpedient to place too implicit a reliance on the theory of an equable and uniform variation of rate, which forms the basis of the preceding investigations, over longer periods than is absolutely unavoidable, and advisable, on the contrary, to limit that assumption in practice to periods as short as the nature of circumstances admits of.

It may, moreover, be held as a general rule, that the dependence which may be placed on the result of any given measurement is inversely proportional to the number of days occupied in the transits from place to place, or elapsed between the epochs of the observations.

It will, of course, often be out of the power of those who undertake the measurements of meridian distances to arrange the conditions under which they are to be performed, or to do more than accept for that purpose such opportunities as chance may place in their way.

In so far, however, as they may have the control over their operations, it would be well for them to arrange that the observations for the determination of the errors and rates before starting should immediately precede the departure from port, and that at the termination of the voyage no unnecessary delay should elapse before those elements were again determined.

The diligent observer will, moreover, of course, avail himself of every opportunity of obtaining the time at intermediate stations; and, in order to break up the measurements into as short links as

possible, will seize every occasion of obtaining new rates when the ship remains sufficiently long at any station to afford time for that process.

Combination of Mouchez's Plan of Correction for Temperature with Tiarks' Method.

We now proceed to explain how, in cases where the fluctuation of temperature during the passage has been considerable, its effects on the performances of the chronometers are to be allowed for. In the present state of chronometric science, and pending the decision of time and experience on the methods proposed by Lieussou and Hartnup, we think that we cannot do wrong in recommending Mouchez's method to the attention of navigators; although perhaps not theoretically perfect, it at any rate possesses three essential properties, which render it suitable for practice at sea, and may cause it to find favour with seamen: the corrections for temperature are derived from the actual sea observations, while the application of them is simple, expeditious, and short.

Adhering in other respects to the notation* previously employed in this chapter, to explain Tiarks' mode of treatment, let θ be the mean daily temperature corresponding to the period of rating, when the rate was a; θ' the corresponding temperature during the period when the rate was subsequently found to be $a + b$; θ_1 the mean daily temperature during the period τ in the partial passage from A to B; θ_2 the mean daily temperature during the period $\tau' - \tau$ in the partial passage from B to C; θ_n the mean temperature for the whole period of the passage from A to K during the interval t.

Then the value of the coefficient of temperature y can be immediately ascertained from the comparison of any two observations for the rate made under favourable circumstances of

* We must claim the indulgence of our readers for some apparent confusion of notation. In discussing Mouchez's method, *ante*, p. 136, we have called the temperature t, because it was necessary subsequently to compare his plan with Lieussou's. In Tiarks' formulæ t is already appropriated to signify time, hence we propose here to use θ (*theta*), the initial letter of the word thermometer. In discussing the system of previous writers it is always advisable to adhere as nearly as possible to their notation, introducing as few changes as possible, which principle has guided us on the present occasion.

considerable differences of temperature, and is given by the formula,

$$y = \frac{b}{\theta - \theta'}$$

There are good grounds for supposing that, in a good chronometer, whose compensation for temperature has been accurately adjusted, the value of y, once carefully determined, remains sensibly constant, while the instrument is in good condition. The diligent observer, who studies his chronometer with the view to obtaining a complete confidence in its performances, will not fail, however, to verify the value of this coefficient, whenever the circumstances of his observation for rate have been propitious for it. When the chronometer is aged, its oils thickened, or partially evaporated, and its movements affected by the petty causes of derangement supervening by time, it may be expected that this coefficient will undergo a sensible alteration as time advances. In the system under discussion the influence of the acceleration is mixed up with, and involved in, that due to the change of temperature; hence, as the performance of the chronometer gradually deteriorates by the lapse of time, the value of the coefficient y may be expected slowly to change.

Again, the researches of Lieussou, and Hartnup's experiments, all tend to show that the correction for, or coefficient of, temperature is not exactly the same in all parts of the thermometric scale (see *ante*, p. 143); the value of y, in fact, determined from time to time by any given observations, in strictness actually corresponds at the time to the mean temperature $\frac{\theta + \theta'}{2}$.

Bearing these facts in mind, the diligent navigator will of course avail himself of every opportunity for verifying the value of the coefficient y, as frequently as possible, and under as varying circumstances of temperature as possible, for each of his chronometers; he himself will then be the best judge what degree of confidence he can bestow on its permanence.

For the correction of the longitude of the ship at sea, the value of the coefficient y previously determined will be no doubt, in general, sufficiently accurate, in cases where it is considered necessary to apply it.

In this case, instead of assuming, as is usually done, the in-

variability of the initial rate a, and obtaining the longitude of the ship from the formula,

$$M = \lambda' - (\lambda + t\,a)$$

we should use the equation,

$$M = \lambda' - \{\lambda + t\,a + t\,(\theta - \theta_1)\,y\}^*$$

on which we shall only further observe, that attention must be paid to the algebraic sign of the coefficient y, or, which comes to the same thing, the correction for change of temperature $t\,(\theta - \theta_1), y$ must be applied according to its actual observed effects in altering the rate.

Again, in a similar manner, the four formulæ given *ante*, pp. 157–8, for the measurement of meridian distances, become by the introduction of the temperature corrections as follows:—

For the whole meridian distance from A to K:

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right) + t\left(\frac{\theta + \theta'}{2} - \theta_n\right) y\right\} \tag{9}$$

From A to B:

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau\,a + \frac{\tau^2}{2\,t}\,b\right) + \tau\left(\frac{\theta + \theta'}{2} - \theta_1\right) y\right\} \tag{10}$$

From B to C:

$$M_2 = \lambda_2 - \left\{\lambda_1 + \left(\overline{\tau' - \tau}\,.\,a + \frac{\overline{\tau' + \tau}\,.\,\overline{\tau' - \tau}}{2\,t}\,b\right) + \overline{\tau' - \tau}\left(\frac{\theta + \theta'}{2} - \theta_2\right) y\right\} \tag{11}$$

From C to K:

$$M_3 = \lambda_3 - \left\{\lambda_2 + \left(\overline{t - \tau'}\,.\,a + \frac{\overline{t + \tau'}\,.\,\overline{t - \tau'}}{2\,t}\,b\right) + \overline{t - \tau'}\left(\frac{\theta + \theta'}{2} - \theta_3\right) y\right\} \tag{12}$$

The example already given at p. 136, is an illustration of the application to practice of formula (9) for the measurement of a

* Recapitulating the notation for the sake of clearness: M is the longitude of the ship sought for; λ the original error of the chronometer on Greenwich mean time at starting on the voyage; λ' the error of the chronometer on time at the ship, determined by observation on the day in question; a the initial daily rate, ascertained before departure, at the mean temperature θ; θ_1 the mean of the mean daily temperatures of the chronometers, observed since leaving port for the time t, elapsed since the epoch, when the rate was a; y the coefficient of temperature previously determined.

meridian distance between two terminal stations. The formulæ are themselves so simple, and their application to numerical calculations so easy, merely involving the use of one additional term, in the correction of the error of the chronometer on local mean time, at the first station, that their further amplification seems superfluous.*

We have only further to observe that it will always be advisable, if possible, to obtain the coefficient y from the actual rate observations of the passage under discussion, combining them together for this purpose in pairs in the most advantageous manner, according to circumstances. If this cannot be done the value of y, derived from previous observations, must needs be employed in lieu.

The actual coefficient of temperature at the period is, however, so much more preferable, that officers should not refuse the trouble of obtaining it if possible. It involves not only the influence of the acceleration then existing, but necessarily depends on that particular range of temperature within which the observations have been made. We have no certainty that this is the case, if a coefficient determined by prior and distant observations is employed. The permanence, or small variation of the coefficient y, affords a test of the goodness of the chronometer.

Comparison of Flinders' and Tiarks' Methods.

We shall now proceed to institute a comparison between the methods of correcting for changes of rate usually adopted by different English navigators.

If we assume that at the epoch of the first observation the rate were a, and that being subsequently found to have changed to $a + b$, we consider the change to have taken place uniformly and in proportion to the time, following the law of an "arithmetic series" (which is the hypothesis adopted by Flinders, King, Owen, and others); then the interval between the epochs being t, the

* We honestly confess that this is rather slurring over a difficulty; we should much have preferred, in accordance with our usual custom, to have illustrated these formulæ by applying them to examples of actual observations. Unfortunately the paucity of sea observations, in which the records of temperature of the chronometers have been carefully made hitherto is such, that data are not accessible, even if existing at all, for this purpose.

daily variation would be $\frac{b}{t}$, and the accumulation of the rate for any partial interval, τ, would be

$$\tau a + \left\{ \frac{b}{t} + \frac{2b}{t} + \frac{3b}{t} + \cdots\cdots + \frac{\tau b}{t} \right\}$$

in which expression it is to be noted that the first term in the series equals the common difference.

Now in any arithmetic series, in which the first term equals the common difference,

$$S = q(n+1)\frac{n}{2}$$

S being the sum of the terms, n their number, and q the first term, or common difference.

Hence our expression above, in a similar manner, becomes,—

$$\begin{aligned} \text{Accumulated rate} &= \tau a + (\tau + 1)\frac{\tau}{2t}b \\ &= \tau a + \frac{\tau^2}{2t}b + \frac{\tau}{2t}b \end{aligned}$$

in which expression the two last terms denote the correction for interpolation for the change of rate, or, as it has been sometimes called, the supplemental error.

Capt. Owen's formula (see Owen on Chronometers, p. 4) is, in principle, identically the same with the above, only with a difference of notation, the factor $\frac{1}{4}$ being also introduced to reduce the correction from seconds of time to minutes of arc.

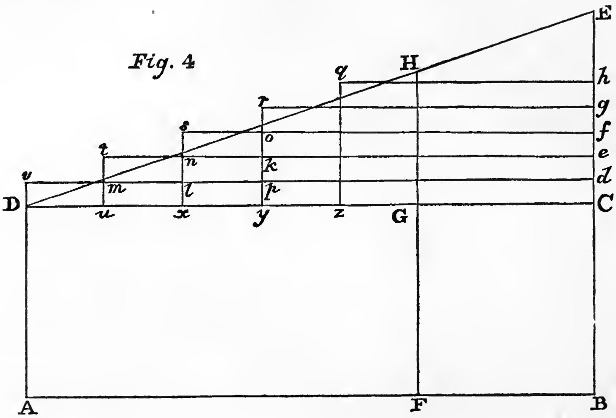
Hence we see that, according to the view adopted by Flinders, King, Owen, and others,

The accumulated rate for any partial interval, τ	$=$	$\tau a + \frac{\tau^2}{2t}b + \frac{\tau}{2t}b$
And according to Tiarks, Bayfield, &c. (*ante*, p. 153) the accumulation of the rate	$=$	$\tau a + \frac{\tau^2}{2t}b$
Hence their difference	$=$	$\frac{\tau}{2t}b$

is the excess of the former over the latter.

A graphic delineation will be useful to show that this dif-

ference is a real excess, and that Tiarks' idea, which we adopt, is the more consistent hypothesis of the two, and affords the more elegant result.



In the annexed figure, let A B or D C $= t$ represent the whole interval between the epochs for which the rates were determined, and let A F or D G $= \tau$ represent the partial interval for which it is proposed to find the correction. Also, let A D, the original rate, $= a$, and C E, the change of rate, $= b$.

Let C E, the change of rate, be divided into t parts, C d, $d\,e$, $e f$, $f g$, &c., each, therefore, equal to $\frac{b}{t}$.

Also, let A B or D C be, in like manner, divided into t parts; each, therefore, equal to unity, since A B $= t$.

Draw the parallels $d\,v$, $e\,t$, $f\,s$, &c., and also the perpendiculars, v D, $t\,u$, $s\,x$, &c.

Now, according to Tiarks' view, the whole accumulation of the rate for the interval D G or τ is represented by the areas A F G D and D G H, in which the supplemental part due to the change of rate, b, is represented by the area of the triangle D G H; and in Flinders' plan, assuming the supplemental part to be the sum of an arithmetic series, in which the first term equals the common difference, the proportion for the first day $= \frac{b}{t} \times 1 =$ the small rectangle D $u\,m\,v$;

That for the second day $= \frac{b}{t} \times 2 =$ the sum of the small rectangles $u\,x\,l\,m$ and $m\,l\,n\,t$;

And similarly for the third day, the sum of the small rectangles, $x\,y\,p\,l$, $l\,p\,k\,n$, and $n\,k\,o\,s$—and so on.

Hence, therefore, the whole supplemental correction for the days τ, evidently equals the sum of all the small rectangles to the left of the line G H, which, again, evidently exceeds the area of the triangle, D G H, by the sum of the small triangles D $m\,v$, $m\,n\,t$, $n\,o\,s$, &c.

Now the area of each of these small triangles obviously equals $\frac{b}{t} \times \frac{1}{2}$, and, therefore, for τ days their sum $= \frac{\tau}{2\,\tau}\,b$; which excess, obtained by a geometric consideration, equals that deduced before from algebraic principles.

We think that, as Tiarks remarks, his formula is more correct, and rather more simple; and it is surely more natural and more consistent to suppose that the accumulation of the rate flows on equably, following the law of the area of a right-angled triangle, than that it proceeds by successive increments corresponding to given increments of time in accordance with the law of an arithmetic series.

It is worthy of note, that the idea of the area of a right-angled triangle is what the latter supposition resolves itself into, if the increment of the time $d\,x$ and the increment of the rate $d\,y$ are both assumed to be indefinitely small, and the limits of x and y respectively are taken from $x = o$ to $x = t$, and from $y = o$ to $y = b$; in which case, by integration, we shall have,—

$$\text{Area of rectangle, or twice area of triangle} \quad = \int_o^t \int_o^b dx \, . \, dy$$

And performing the integration,

$$\text{Twice area of triangle} \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad t\,b$$

$$\left.\begin{array}{l}\text{Hence area of triangle, or whole accumula-}\\ \quad\text{tion due to the variation of rate for the}\\ \quad\text{time } t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\} = \frac{t\,b}{2}$$

Whence, as before,

$$\text{Accumulation for the partial interval } \tau \ldots\ldots \quad = \frac{\tau^2}{2\,t}\,b$$

See Appendix to Forster's "Voyage," vol. ii. p. 226; "Voyage of Adventure and Beagle," Appendix to vol. ii. p. 320; Owen on Longitudes, Explanation, p. 4.

Methods of Vincendon Dumoulin, Coupvent Desbois, and Charles Ploix.

Besides the methods of chronometric treatment, explained in this and the preceding chapter, proposed by De Cornulier, Lieussou, Mouchez, Hartnup, and Tiarks, the subject has also received further illustration from the recent writings of MM. Vincendon Dumoulin, Coupvent Desbois, and Charles Ploix. The two former gentlemen are the joint authors of a hydrographical memoir, on the voyage of the Capricieuse, and since reprinted in the "Recherches Chronométriques," pp. 177–207, which we have already had occasion to quote in these pages (*ante, p.* 99); M. Ploix's memoir is also given in the same work.*

Although their mode of proceeding is somewhat different, the principle of their two methods is essentially the same. Instead of employing only the rates of departure and arrival, as is done by Tiarks, in measuring meridian distances, they propose to utilise for this purpose all the rates which may have been obtained during the voyage, at any of the intermediate points at which the ship may have touched.

The equation of rate of a chronometer may always be written under the form

$$y = \mathrm{F}(x)$$

in which x represents the time elapsed since some given epoch, and y is the ordinate of the curve of the chronometer. The curve of a chronometer being defined to be the locus of a point whose abscissa is a line representing the time elapsed since a given epoch, and ordinate a line representing the actual rate of the chronometer, at that moment.

If only the rates of departure and arrival are employed, the curve of the chronometer becomes a straight line, and its equation is

$$y = a + b\,x$$

If several determinations of the rate have been obtained, each one furnishes a value of the ordinate y, and defines a point in the curve, whose equation may then be written under the form

$$y = a + b\,x + c\,x^2 + d\,x^3 + \&c.$$

in which there will be as many coefficients a, b, c, &c., to

* Sur le calcul des longitudes determinés au moyen des chronomètres; par M. Charles Ploix, "Ingénieur Hydrographe." "Recherches Chronométriques," p. 209.

determine, as the observations have furnished data to clear up the question. The whole problem then consists in the calculation of these coefficients; the definitive equation being once obtained, by putting for x_1 any date whatever (reckoned from the epoch of departure) the value of y that we deduce from it, will be the rate of the watch at that date. We should have thus, the rate for every day of the passage.

The whole accumulation of the rate of a chronometer in the interval between any two observations is, in any case, the area comprised between the axis of x, the curve of the chronometer, and the two ordinates corresponding to the epochs of the observations in question. When only two observations are employed, the curve of the chronometer, as we before remarked, becomes a straight line, and this area is the rectangle A B E D (*ante*, p. 152), as we have already shown while discussing Tiarks' method. In any other case, all the observed rates during the voyage being employed for the solution of the question, the area is the definite integral $\int y.dx$, which may be determined by the application of the principles of the integral calculus.

On this matter MM. Vincendon Dumoulin and Coupvent Desbois remark, "Although we do not know the nature of the curve of a chronometer, we know that it must pass through certain known points. Now to determine the form of the function $y = f(x)$, we can employ the means furnished by the calculus of differences, which geometers use to determine the value of a function corresponding to a certain value of the variable, without knowing the nature of this function, provided that it be intermediate between the given values of the function, corresponding to certain values of the variable."

These gentlemen then proceed to show, how the formulæ of interpolation, proposed by Lagrange, may be employed for the solution of this problem. M. Ploix, in a subsequent memoir, given in the "Recherches Chronométriques," while expressing his general concurrence with these writers' views, points out how the treatment of the question may be somewhat simplified.

The mathematical discussions in these memoirs, interesting and elegant though they be, are somewhat too abstruse for these pages, and we do not therefore consider it necessary to enter more fully into their consideration, or do more than take passing

notice of them, and commend them to the attention of the scientific student.

We are the more reconciled to this course, because we gravely doubt whether, as a matter of practice, the application of these principles is an improvement on the plan recommended for adoption in these pages, viz. to employ Tiarks' formulæ, combined with Mouchez's for correcting for temperature,—to break up the meridian distances into as short links as possible,—and to obtain the rates as often as possible. Every meridian distance depending on two observed rates obtained before departure, and after arrival, is then complete within itself, as one of the connecting links in the chain of "differences of meridians," found chronometrically during the voyage.

No doubt the assumption, that the curve of the chronometer in the interval is virtually a straight line, involves some errors, for probably in all cases it is really a sinuous curve, having possibly many points of flexure; hence the calculation of the area of the figure, which represents the accumulated rate during the passage, on the supposition that it is rectilinear, whereas in truth one of its containing sides is a curve, must be deficient in accuracy. The errors thus introduced may, however, be restricted within very narrow limits, by taking the intervals as small as possible, and probably a considerable part of them may be destroyed, by the application of the correction for temperature (as previously recommended), of which these errors themselves would seem to be chiefly a function. The residual error would then probably be extremely small. In other respects, the plan proposed by these authors would probably, in practice, involve the employment of rate observations extending over a long period. MM. V. Dumoulin and C. Desbois themselves, give an example in illustration of their method, in which the interval between the extreme observations exceeds sixty days. No doubt, in the measurement of "meridian distances" the shorter the interval the better, and as Mouchez truly observes, "the last rate obtained is always the best."* Tiarks' formulæ, moreover, are so simple and so well adapted to practice, that their use ought not to be displaced on light grounds, and hence, for the present at least, we recommend that they be adhered to.

* "Recherches Chronométriques," p. 273.

CHAPTER VIII.

VARIOUS EXAMPLES OF THE COMPUTATION OF MERIDIAN DISTANCES, ILLUSTRATING THE APPLICATION OF TIARKS' FORMULÆ GIVEN IN THE PRECEDING CHAPTER.

We shall now proceed to illustrate the formulæ for the measurement of meridian distances given in the preceding chapter by some numerical examples, exhibiting the varieties of the different cases which can arise in actual practice.

In the arrangement of the examples we shall suppose, in all cases, that three chronometers have been employed in determining the meridian distance: this number will be conveniently suitable to the width of the page, in so far as typographical arrangements are concerned, and at the same time, without being unnecessarily diffuse, and without distracting the attention of the student by a needless amplification of numerical details, it will probably be found to afford sufficient variety in illustration of the application of the corrections for rate, and of the general manipulation of the formulæ. Each example thus exhibiting the results by three chronometers, is to be taken as a type of the computations required for any number whatever, the reduction of results by extra ones being made on precisely similar principles, and only involving a slight addition for each chronometer to the numerical calculations already made.

Case I.—Meridian distance from A to K. Errors and rates determined at each place.

Example 1. By single-altitude A.M. observations on Jan. 1st and 7th, 1844, at Ross Bank Observatory, Hobarton, Van Diemen's Land, the errors and rates of chronometers Z, C, and I, at the mean epoch, Jan. $3^{d}\cdot875$, were,—

	Errors.				Rates.
		h	m	s	s
Chron. Z	—	9	53	17·58	+ 0·13
C	—	8	47	24·63	+ 10·41
I	—	10	50	55·83	— 4·29

On arrival at Sydney, New South Wales, by equal-altitude observations at Garden Island on Jan. 26th and Feb. 2d, the errors and rates at the mean epoch, Jan. $29^{d}\cdot5$, were found to be,—

	Errors.			Rates.
	h	m	s	s
Chron. Z	− 10	9	1·02	− 1·14
C	− 8	58	37·94	+ 10·26
I	− 11	8	32·29	− 5·07

From these data the meridian distance is required.

The rates of the chronometers having been determined at each place, the meridian distance is to be obtained by formula (1).

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\}$$

In which expression λ' refers to the errors at Sydney, and λ to those at Hobarton; also t, the interval between the epochs, is $25^{d}\cdot625$, and the mean rates $\left(a + \frac{b}{2}\right)$ for the several chronometers are,—

	s
Z	− 0·50
C	+ 10·33
I	− 4·68

Whence we have,

	Z.	C.	I.
	h m s	h m s	h m s
Chron. slow on Hobarton mean time, Jan. $3^{d}\cdot875$, or λ	− 9 53 17·58	−8 47 24·63	−10 50 55·83
Accumulated rate or $t\left(a + \frac{b}{2}\right)$	−12·81	+4 24·70	−1 59·92
Chron. slow on Hobarton mean time, Jan. $29^{d}\cdot5$, or $\lambda + t\left(a + \frac{b}{2}\right)$	− 9 53 30·39	−8 42 59·93	−10 52 55·75
Do. do. at Sydney, or λ'	−10 9 1·02	−8 58 37·94	−11 8 32·29
Meridian distance ..	− 0 15 30·63	−0 15 38·01	− 0 15 36·54

Hence, as the meridian distance by these three chronometers, we have,—

	h	m	s
Z	− 0	15	30·63
C			38·01
I			36·54
Mean ..	− 0	15	35·06

Example 2. By equal-altitude observations at Moulmain (Government Wharf), on Jan. 10th and 15th, 1853, the errors and rates of chronometers A, C, and Y, at the mean epoch, Jan. $12^{d}\cdot5$, were as follows:—

	Errors.			Rates.
	h	m	s	s
Chron. A	− 6	7	14·80	− 5·34
C	− 6	21	24·56	+ 2·16
Y	− 0	17	55·06	+ 5·96

Subsequently the errors and rates at Rangoon at the mean epoch, Jan. $22^{d}\cdot5$, by equal altitudes at the Commodore's Wharf on Jan. 19th and 26th, were,—

	Errors.			Rates.
	h	m	s	s
Chron. A	− 6	2	11·92	− 4·07
C	− 6	15	11·02	+ 2·67
Y	− 0	10	54·27	+ 7·89

Here, as before, the rates having been obtained at each place, the meridian distance is to be obtained by formula (1).

$$M = \lambda' - \left\{ \lambda + t\left(a + \frac{b}{2}\right) \right\}$$

where λ refers to the errors at Moulmain, and λ' to those at Rangoon, while $t = 10^{d}\cdot0$.

Also, for the respective chronometers, the accumulated correction for the rate, or $t\left(a + \frac{b}{2}\right)$ will be,—

	s
Chron. A	− 47·00
C	+ 24·20
Y	+ 69·20

Whence we have,

	A.	C.	Y.
	h m s	h m s	h m s
Error of chron. on Moulmain mean time, or λ, on Jan. $12^{d}\cdot5$	−6 7 14·80	−6 21 24·56	−0 17 55·06
Accumulated rate, or $t\left(a + \frac{b}{2}\right)$	−47·00	+24·20	+1 9·20
Error of chron. on Moulmain mean time, on Jan. $22^{d}\cdot5$, or $\lambda + t\left(a + \frac{b}{2}\right)$	−6 8 1·80	−6 21 00·36	−0 16 45·86
Error of chron. on Rangoon mean time, Jan. $22^{d}\cdot5$, or λ'	−6 2 11·92	−6 15 11·02	−0 10 54·27
	+0 5 49·88	+0 5 49·34	+0 5 51·59

Therefore, by these chronometers we have, as the result for the meridian distance,—

			h	m	s
Chron. A		+	0	5	49·88
C					49·34
Y					51·59
Mean	..	+	0	5	50·27

Case II.—Meridian distances between A, B, C, and K. Both errors and rates determined at the terminal stations A and K; while errors on time only were ascertained at the intermediate stations, B and C.

Example 1. In a voyage between La Guayra and Carthagena in the West Indies, calling on the way at Porto Cabello and Curaçoa, the following observations having been made, the respective meridian distances are required.

By observations at La Guayra on May 22d and 28th, the errors and rates of chronometers F, M, and P, at the mean epoch, May $24^{d}\cdot885$, were as follows:—

	Errors.				Rates.
		h	m	s	s
Chron. F	+	4	33	7·80	− 0·77
M	+	4	0	17·40	+ 4·54
P	+	5	9	43·70	+ 1·47

On arrival at Porto Cabello, the errors on mean time at the place on June $5^{d}\cdot870$, were ascertained to be,—

		h	m	s
Chron. F	+	4	37	15·80
M	+	4	5	31·28
P	+	5	14	13·38

Passing on to Curaçoa, the errors on June $12^{d}\cdot890$ were,—

		h	m	s
Chron. F	+	4	40	59·20
M	+	4	9	55·53
P	+	5	18	3·24

And finally, on arrival at Carthagena, observations on the 25th and 29th June gave the errors and rates at the mean epoch, June 27^{d}, as follows:—

	Errors.	Rates.
	h m s	s
Chron. F	+ 5 7 23·55	− 0·85
M	+ 4 37 47·98	+ 5·90
P	+ 5 44 34·42	− 0·30

Here, as elements of the calculation, we have for the intervals between the epochs of the observations, $t = 33^{d}\cdot139$ (between La Guayra and Carthagena); $\tau = 11^{d}\cdot988$ (between La Guayra and Porto Cabello); and $\tau' = 19^{d}\cdot010$ (between La Guayra and Curaçoa).*

Also, for the several chronometers, the mean rate, $a + \frac{b}{2}$, and the variation of the rate, b, is as follows:—

	Mean Rate $\left(a + \frac{b}{2}\right)$.	Variation of Rate, b.
	s	s
Chron. F	− 0·81	− 0·08
M	+ 5·22	+ 1·36
P	+ 0·58	− 1·77

Then, first, for the meridian distance between the terminal stations La Guayra and Carthagena, where the rates were determined, we have, by formula (1),—

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\}$$

and solving numerically,

	F.	M.	P.
	h m s	h m s	h m s
Error of chron. on La Guayra M.T., or λ, on May 25th ..	+4 33 7·80	+4 0 17·40	+5 9 43·70
Accumulated rate $t\left(a + \frac{b}{2}\right)$	−26·84	+2 52·98	+19·22
Error at La Guayra, June 27th, or $\lambda + t\left(a + \frac{b}{2}\right)$	+4 32 40·96	+4 3 10·38	+5 10 2·92
Error at Carthagena, June 27th, or λ'	+5 7 23·55	+4 37 47·98	+5 44 34·42
	+0 34 42·59	+0 34 37·60	+0 34 31·50

Whence we have,

Meridian distance La Guayra to Carthagena.

	h m s
Chron. F	+ 0 34 42·59
M	37·60
P	31·50
Mean ..	+ 0 34 37·23

* Applying to the several apparent intervals the corrections for the approximate difference of longitude, viz. $0^{d}\cdot024$, $0^{d}\cdot003$, and $0^{d}\cdot005$.

Secondly, for the partial measurement between La Guayra and Porto Cabello we have, by formula (2),—

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau\, a + \frac{\tau^2}{2\,t}\, b\right)\right.$$

in which λ refers to the errors at La Guayra, the place left, and λ_1 to those at Porto Cabello, the place arrived at.

Also the factor $\frac{\tau^2}{2\,t}$ is [0·336124],* and for the several chronometers we have,—

	$\tau\, a$	$\frac{\tau^2}{2\,t}\, b$	$\left(\tau\, a + \frac{\tau^2}{2\,t}\, b\right)$
	s	s	s
Chron. F	− 9·23	− 0·17	− 9·40
M	+ 54·43	+ 2·95	+ 57·38
P	+ 17·62	− 3·84	+ 13·78

And applying the corrections,—

	F.	M.	P.
	h m s	h m s	h m s
Error of chron. on La Guayra M.T., or λ, on May 25th ..	+4 33 7·80	+4 0 17·40	+5 9 43·70
Accumulated rate $\left(\tau a + \frac{\tau^2}{2\,t}\, b\right)$	−9·40	+57·38	+13·78
Error at La Guayra, June 6th, or (λ + cor.)	+4 32 58·40	+4 1 14·78	+5 9 57·48
Error at Porto Cabello, June 6th, or λ_1	+4 37 15·80	+4 5 31·28	+5 14 13·38
	+0 4 17·40	+0 4 16·50	+0 4 15·90

Hence, for the meridian distance between La Guayra and Porto Cabello, we have,—

	h	m	s
Chron. F	+ 0	4	17·40
M			16·50
P			15·90
Mean ..	+ 0	4	16·60

Thirdly, for the intermediate measurement between Porto Cabello and Curaçoa, by formula (3), we have,—

$$M_2 = \lambda_2 - \left\{\lambda_1 + \left(\overline{\tau' - \tau} \,.\, a + \frac{\overline{\tau' + \tau} \,.\, \overline{\tau' - \tau}}{2\,t}\, b\right)\right\}$$

where λ_1 refers to Porto Cabello and λ_2 to Curaçoa. Also $(\tau' - \tau) = 7^{d}{\cdot}022$, and the factor $\frac{\overline{\tau' + \tau} \,.\, \overline{\tau' - \tau}}{2\,t} = [0{\cdot}516425]$.

* A number enclosed between brackets, as above, signifies the logarithm of the numerical factor under discussion.

Whence, for the several chronometers, we have,—

	$\overline{\tau'-\tau}\,.\,a$;	$\dfrac{\overline{\tau'+\tau}\,.\,\overline{\tau'-\tau}}{2t}\,b$;	$\left\{\overline{\tau'-\tau}\,.\,a+\dfrac{\overline{\tau'+\tau}\,.\,\overline{\tau'-\tau}}{2t}\,b\right\}$
	s	s	s
Chron. F	− 5·41	− 0·26	− 5·67
M	+ 31·88	+ 4·47	+ 36·35
P	+ 10·32	− 5·81	+ 4·51

And proceeding as before,—

	F.	M.	P.
	h m s	h m s	h m s
Error of chron. on Porto Cabello M.T., on June 6th, or λ_1	+4 37 15·80	+4 5 31·28	+5 14 13·38
Accumulated rate (as above)	−5·67	+36·35	+4·51
Error at Porto Cabello, June 13th, or (λ_1 + cor.)	+4 37 10·13	+4 6 7·63	+5 14 17·89
Error at Curaçoa, June 13th, or λ_2	+4 40 59·20	+4 9 55·53	+5 18 3·24
	+0 3 49·07	+0 3 47·90	+0 3 45·35

Therefore, for the meridian distance between Porto Cabello and Curaçoa, we obtain,—

		h	m	s
Chron. F	+	0	3	43·07
M				47·90
P				45·35
Mean ..	+	0	3	47·44

Lastly, for the final link between Curaçoa and Carthagena, formula (4) gives us,—

$$M_3 = \lambda' - \left\{\lambda_2 + \left(\overline{t-\tau'}\,a + \frac{\overline{t+\tau'}\,.\,\overline{t-\tau'}}{2t}\,b\right)\right\}$$

in which λ_2 refers to Curaçoa and λ' to Carthagena.

Also $t-\tau' = 14^{d}\cdot129$, and the factor $\dfrac{\overline{t+\tau'}\,.\,\overline{t-\tau'}}{2t} = [1\cdot045988]$.

Likewise for the several chronometers we have,—

	$\overline{t-\tau'}\,a$;	$\dfrac{\overline{t+\tau'}\,.\,\overline{t-\tau'}}{2t}\,b$;	$\left\{\overline{t-\tau'}\,a+\dfrac{\overline{t+\tau'}\,.\,\overline{t-\tau'}}{2t}\,b\right\}$
	s	s	s
Chron. F	− 10·88	− 0·89	− 11·77
M	+ 64·15	+ 15·12	+ 79·27
P	+ 20·77	− 19·68	+ 1·09

Whence, as before,—

	F.	M.	P.
	h m s	h m s	h m s
Error of chron. on Curaçoa M.T., on June 13th, or λ_2..	+4 40 59·20	+4 9 55·53	+5 18 3·24
Accumulated rate (as above)	−11·77	+1 19·27	+1·09
Error at Curaçoa on the 27th, or (λ_2 + cor.)	+4 40 47·43	+4 11 14·80	+5 18 4·33
Error at Carthagena on the 27th, or λ'..............	+5 7 23·55	+4 37 47·98	+5 44 34·42
	+0 26 36·12	+0 26 33·18	+0 26 30·09

Therefore, for the meridian distance from Curaçoa to Carthagena, we have,—

		h	m	s
Chron.	F	+ 0	26	36·12
	M			33·18
	P			30·09
Mean ..		+ 0	26	33·13

Finally, collecting the several partial results for examination and comparison, we have,—

Meridian distance,	F.	M.	P.
	h m s	h m s	h m s
I. La Guayra to Porto Cabello	+0 4 17·40	+0 4 16·50	+0 4 15·90
II. Porto Cabello to Curaçoa	+0 3 49·07	+0 3 47·90	−0 3 45·35
III. Curaçoa to Carthagena ..	+0 26 36·12	+0 26 33·18	+0 26 30·09
Therefore, meridian distance from La Guayra to Carthagena, by sum of partial measurements	+0 34 42·59	+0 34 37·58	+0 34 31·34
Do. by direct measurement ..	+0 34 42·59	+0 34 37·60	+0 34 31·50

The accordance of the final results by the partial and direct measurements shows that, whatever may be the merits of the measurements, in so far as particular chronometers are concerned, the formulæ here employed deal with the observations in a uniform and systematic manner, and at any rate yield consistent results.

Example 2. By equal-altitude observations at Cape Upstart (north-east coast of Australia) on May 2d and 6th, 1844, the errors and rates of chronometers A, B, and C, at the mean epoch, May 4^d, were as follows:—

	Errors.			Rates.
	h	m	s	s
Chron. A	− 9	46	16·20	− 0·79
B	− 9	49	47·85	− 0·46
C	− 8	28	26·55	+ 10·94

On arrival at Lizard Island on May 12th, equal-altitude observations gave the errors of the chronometers on local mean time,—

	h	m	s
Chron. A	− 9	37	12·22
B	− 9	40	37·92
C	− 8	17	48·82

And finally, equal-altitude observations at Sir Charles Hardy's Islands, on May 21st and 25th, gave the errors and rates at the mean epoch, May 23^d, as under,—

	Errors.			Rates.
	h	m	s	s
Chron. A	− 9	29	28·82	− 1·39
B	− 9	32	58·72	− 2·29
C	− 8	7	56·27	+ 10·83

From these data the several meridian distances between the stations are to be obtained.

Here $t = 19^d{\cdot}00$ (the interval between the observations at the terminal stations), and $\tau = 8^d{\cdot}00$ (the interval to the intermediate observation at Lizard Island).

Also $\frac{\tau^2}{2t} = [0{\cdot}226396]$ and $\frac{\overline{t+\tau}\,.\,\overline{t-\tau}}{2t} = [0{\cdot}872973]$

while we have for the several chronometers,—

	Mean Rate $\left(a + \frac{b}{2}\right)$	Variation of Rate (b).
	s	s
Chron. A	− 1·09	− 0·60
B	− 1·37	− 1·83
C	+ 10·88	− 0·11

Then, first, for the meridian distance between the terminal stations, Cape Upstart to Sir C. Hardy's Islands, we have, by formula (1),

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\}$$

And applying the formula,

	A.	B.	C.
	h m s	h m s	h m s
Error of chron. on C. Upstart mean time, May 4^d, or λ ..	−9 46 16·20	−9 49 47·85	−8 28 26·55
Cor. for rate $t\left(a+\frac{b}{2}\right)$	−20·71	−26·03	+3 26·72
Error at C. Upstart, May 23^d, or $\lambda + t\left(a+\frac{b}{2}\right)$	−9 46 36·91	9 50 13·88	−8 24 59·83
Error at Sir C. Hardy's Isles, or λ', on May 23^d	−9 29 28·82	9 32 58·72	−8 7 56·27
	+0 17 8·09	+0 17 15·16	+0 17 3·56

Whence we have,—

Meridian distance Cape Upstart to Sir C. Hardy's Islands,—

	h	m	s
Chron. A	+0	17	8·09
B			15·16
C			3·56
Mean ..	+0	17	8·93

Again, for the partial measurement from Cape Upstart to Lizard Island, by formula (2),—

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau a + \frac{\tau^2}{2t} b\right)\right\}$$

And solving numerically,

	A.	B.	C.
	h m s	h m s	h m s
Error of chron. on C. Upstart mean time, May 4^d, or λ ..	−9 46 16·20	−9 49 47·85	−8 28 26·55
Cor. for rate $\left(\tau a + \frac{\tau^2}{2t} b\right)$	−7·33	−6·76	+1 27·33
Error on May 12^d..........	−9 46 23·53	−9 49 54·61	−8 26 59·22
Error at Lizard Island, or λ_1	−9 37 12·22	−9 40 37·92	−8 17 48·82
	+0 9 11·31	+0 9 16·69	+0 9 10·40

Therefore, for the meridian distance, Cape Upstart to Lizard Island, we have,—

	h	m	s
Chron. A	+0	9	11·31
B			16·69
C			10·40
Mean ..	+0	9	12·60

Lastly, from Lizard Island to Sir C. Hardy's Islands, by formula (4),*

$$M_3 = \lambda' - \left\{\lambda_2 + \left(\overline{t - \tau'}\, a + \frac{\overline{t + \tau'} \,.\, \overline{t - \tau'}}{2t}\, b\right)\right\}$$

	A.	B.	C.
	h m s	h m s	h m s
Error of chron. on Lizard Island mean time, May 12d, or λ_2	−9 37 12·22	−9 40 37·92	−8 17 48·82
Cor. for rate, or $\overline{t - \tau'}\, a + \frac{\overline{t + \tau'} \,.\, \overline{t - \tau'}}{2t}\,.\, b$..	−13·17	−18·72	+1 59·52
Error on May 23^d, or (λ_2 + cor.)	−9 37 25·39	−9 40 56·64	−8 15 49·30
Error at Sir C. Hardy's Isles, or λ'	−9 29 28·82	−9 32 58·72	−8 7 56·27
	+0 7 56·57	+0 7 57·92	+0 7 53·03

Hence, for the meridian distance, Lizard Island to Sir Charles Hardy's Islands,—

	h	m	s
Chron. A	+ 0	7	56·57
B			57·92
C			53·03
Mean ..	+ 0	7	55·84

Collating the results as before, for the purpose of comparison and verification, we have,—

Meridian distance,	A.	B.	C.
	h m s	h m s	h m s
I. Cape Upstart to Lizard Island	+0 9 11·31	+0 9 16·69	+0 9 10·40
II. Lizard Island to Sir C. Hardy's Islands	+0 7 56·57	+0 7 57·92	+0 7 53·03
Sum ..	+0 17 7·88	+0 17 14·61	+0 17 3·43
The same by direct measurement	+0 17 8·09	+0 17 15·16	+0 17 3·56

Which results, like those in the preceding example, exhibit a satisfactory degree of accordance.

* We employ formula (4) in preference to (3), (see *ante*, p. 158), because it is the one connecting the last intermediate with the final station (C with K). Formula (3), connecting the intermediate stations (B and C), would serve equally well, if we consider t to become τ', and treat the final as a second intermediate station. In fact, as will be observed, the two formulæ are identical in form.

The preceding examples, illustrating Cases I. and II., afford instances of all the varieties which can arise in practice in what we may term *perfect* cases of the measurement of meridian distances; that is, cases in which, observations for the determination of the rates having been made both at the commencement and termination of the voyage, we have, in point of theory, the means of ascertaining the indication of the chronometers at any moment, and hence of obtaining results for differences of longitude, theoretically perfect, in so far as the formulæ on which they depend are concerned; although, of course, liable practically to many imperfections, from defects of observation, and the mechanical irregularities to which chronometers are constitutionally prone.

We now pass on to the consideration of the *imperfect* cases, in which the incompleteness of the data relative to our knowledge of the rates, and consequent ignorance of the state of the chronometer throughout the period embraced by the observations, prevent our arriving at more than an approximate solution, liable subsequently to be rejected, modified, or amended, when circumstances arise more favourable to a perfect determination.

Case III.—Approximate determination of meridian distance between A and K, the rate having been found at one of the stations only.

Example. By observations at Hong Kong (at Dent's Wharf), on Nov. 10th and 24th, 1850, the errors and rates of chronometers Z, M, and A, at the mean epoch, Nov. 17^d, were,—

	Errors.			Rates.
	h	m	s	s
Chron. Z	− 7	44	12·89	+ 0·40
M	− 7	59	52·29	+ 1·28
A	− 8	20	22·29	− 6·08

On arrival at Shanghai, on Dec. 10^d, the errors of the chronometers were again determined (at Beale's House) as follows:—

	h	m	s
Chron. Z	− 8	13	23·26
M	− 8	28	30·26
A	− 8	51	17·76

From these data we propose to determine the approximate meridian distance.

For this purpose our formula is,—

$$M = \lambda' - (\lambda + t\,a),^*$$

in which λ refers to the errors at Hong Kong, and λ' to those at Shanghai; also t, the interval between the epochs to which the errors refer, is 23^d.

Therefore we have,—

	Z.	M.	A.
	h m s	h m s	h m s
Error of chron. at Hong Kong, Nov. 17^d, or λ	−7 44 12·89	−7 59 52·29	−8 20 22·29
Cor. for rate ($t\,a$)............	+9·20	+29·44	−2 19·84
Error on Dec. 10^d, or ($\lambda + t\,a$)	−7 44 3·69	−7 59 22·85	−8 22 42·13
Error at Shanghai, or λ'......	−8 13 23·26	−8 28 30·26	−8 51 17·76
	−0 29 19·57	−0 29 7·41	−0 28 35·63

Whence, for the meridian distance from Hong Kong to Shanghai, we have, by

	h	m	s
Chron. Z —	0	29	19·57
M		29	7·41
A		28	35·63

which results exhibit a considerable divergence from one another (indicating that at least some of the chronometers have altered their rates), and therefore could not be considered at all satisfactory.

Results obtained in this manner, the rate being only determined at the commencement of the voyage, can only be considered as first approximations, to be subsequently confirmed or amended, when further observations show whether the rate, *a*, on which they depend, has remained constant or undergone a change during the voyage.

A similar remark applies to cases in which the measurement depended on a rate obtained at the termination of the voyage (circumstances not having permitted that element to be obtained before its commencement), since there would be no evidence to show that the subsequent rate had not changed during the voyage, and become materially different from its value before starting.

If, however, the difference of longitude between the two extreme stations should be accurately known, it may be made subservient to the correction of the relative longitudes of intermediate points.

* See *ante*, p. 156.

Case IV.—Rate determined at one station only. Difference of longitude between extreme stations assumed to be known, and applied to correct the relative longitudes of intermediate positions.

Example. Suppose that, on the occasion quoted in the last example, observations for time *only* had been made at Amoy, as follows :—

Nov. 30^d, 1850. By observations at the Cornwallis Rocks, Amoy, the errors of the chronometers were,—

		h	m	s
Chron. Z	—	7	59	56·98
M	—	8	15	23·85
A	—	8	37	24·73

And also that we assume that the true difference of longitude between the places of observation at Hong Kong and Shanghai is,—

$$- 0^h \quad 29^m \quad 17^s{\cdot}10$$

we proceed to apply this to the correction of the position of the intermediate station at Amoy.

By formula (5), the errors of the chronometers λ and λ' at the two stations, and M, their difference of meridians, being known, the *mean sea rate*,—

$$a + \frac{b}{2} = \frac{(\lambda' - \lambda) - M}{t}$$

Hence, for the several chronometers, we have,—

	Z.	M.	A.
	h m s	h m s	h m s
Error at Shanghai, or λ'	—8 13 23·26	—8 28 30·26	—8 51 17·76
Error at Hong Kong, or λ	—7 44 12·89	—7 59 52·29	—8 20 22·29
$\lambda' - \lambda$	—0 29 10·37	—0 28 37·97	—0 30 55·47
M	—0 29 17·10	—0 29 17·10	—0 29 17·10
$(\lambda' - \lambda) - M$	+0 0 6·73	+0 0 39·13	—0 1 38·37

Dividing by $t = 23$,

$$a + \frac{b}{2} = +0^s{\cdot}29 \qquad +1^s{\cdot}70 \qquad -4^s{\cdot}28$$

And by formula (6),—

$$b = 2\left(\overline{a + \frac{b}{2}} - a\right)$$

Whence, for the respective chronometers, the values of b, the variation of the rate during the voyage, are,—

		s
Chron. Z	—	0·22
M	+	0·84
A	+	3·60

Then, for the meridian distance between Hong Kong and Amoy, by formula (2),—

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau a + \frac{\tau^2}{2t} b\right)\right\}$$

Here, $\tau = 13^d$ and $\frac{\tau^2}{2\tau} = [0·565128]$.

Also, for the several chronometers, we obtain,—

	τa	$\frac{\tau^2}{2t} b$	$\tau a + \frac{\tau^2}{2t} b$
	s	s	s
Chron. Z	+ 5·20	— 0·81	+ 4·39
M	+ 16·64	+ 3·09	+ 19·73
A	— 79·04	+ 13·23	— 65·81

And solving the formula numerically,—

	Z.	M.	A.
	h m s	h m s	h m s
Error of chron. at Hong Kong, Nov. 17^d, or λ	−7 44 12·89	−7 59 52·29	−8 20 22·29
Cor. for rate $\left(\tau a + \frac{\tau^2}{2t} b\right)$	+4·39	+19·73	−1 5·81
Error on November 30^d, or $\left\{\lambda + \left(\tau a + \frac{\tau^2}{2t} b\right)\right\}$	−7 44 8·50	−7 59 32·56	−8 21 28·10
Error at Amoy, or λ_1	−7 59 56·98	−8 15 23·85	−8 37 24·73
	−0 15 48·48	−0 15 51·29	−0 15 56·63

Hence, for the meridian distance, Hong Kong to Amoy, we have,—

		h	m	s
Chron. Z	—	0	15	48·48
M				51·29
A				56·63
Mean ..	—	0	15	52·13

In a similar manner the meridian distance from Amoy to Shanghai could be determined, by substituting in formula (4),—

$$M_3 = \lambda' - \left\{\lambda_2 + \left(\overline{t - \tau'}\, a + \frac{\overline{t + \tau'}\,.\,\overline{t - \tau'}}{2\,t}\, b\right)\right\}$$

the values of b previously ascertained; whence we should obtain, on proceeding with the computation,*—

		h	m	s
Chron. Z		− 0	13	28·56
M				25·78
A				21·12
Mean	..	− 0	13	25·15

Comparing this with the previous measurement from Hong Kong to Amoy, their sum equals $-0^h\ 29^m\ 17^s{\cdot}28$, which (small errors excepted) is the same as the assumed value of the difference of longitude of the terminal stations. The same fact would be observable if we compared together the individual results of each chronometer separately.

Hence we see, that by this mode of proceeding the several partial results are all consistent with the primary assumption that the difference of longitude between the terminal stations has a particular value; and although the partial results of any given measurements by different chronometers may differ from one another, yet that the final sum of their several parts, whether taken collectively or separately, all agree.

Another modification of this problem arises when the rate has been determined at the final station only, the error alone having been ascertained at the initial station.

Example. In the preceding example let it be supposed that the error alone was determined before starting, as follows:—

Hong Kong, Nov. 24^d, 1850.

	h	m	s
Chron. Z	− 7	44	10·06
M	− 7	59	43·36
A	− 8	21	4·86

* In this case $\overline{t - \tau'} = 10$, and the factor $\dfrac{\overline{t + \tau'}\,.\,\overline{t - \tau'}}{2\,t} = [0{\cdot}893545]$.

On arrival at Amoy, on Nov. 30^d, the errors (as before) were found to be,—

	h	m	s
Chron. Z	− 7	59	56·98
M	− 8	15	23·85
A	− 8	37	24·73

and, finally, on reaching Shanghai, by observations on Dec. 10th and 18th, the errors and rates at the mean epoch, Dec. 14^d, were found to be,—

	Errors. h	m	s	Rates. s
Chron. Z	− 8	13	21·91	+ 0·34
M	− 8	28	22·29	+ 1·99
A	− 8	51	25·85	− 2·02

Also, as before, let the difference of longitude between the terminal stations be assumed as

$$0^h\ 29^m\ 17^s{\cdot}10$$

and, consistently with this supposition, let us proceed to determine the relative longitudes of intermediate points.

The observations at the terminal station, Shanghai, afford us the respective values of the final rate $(a + b)$.

And by formula (5),—

$$\text{The } \textit{mean sea rate} \left(a + \frac{b}{2}\right) = \frac{(\lambda' - \lambda) - M}{t}$$

and solving numerically for the several chronometers,—

	Z. h m s	M. h m s	A. h m s
Error at Shanghai, or λ'	−8 13 21·91	−8 28 22·29	−8 51 25·85
Error at Hong Kong, or λ	−7 44 10·06	−7 59 43·36	−8 21 4·86
$(\lambda' - \lambda)$	−0 29 11·85	−0 28 38·93	−0 30 20·99
M	−0 29 17·10	−0 29 17·10	−0 29 17·10
$(\lambda' - \lambda) - M$	+0 0 5·25	+0 0 38·17	−0 1 3·89

and dividing by $t = 20^d$,—

$$a + \frac{b}{2} = +\,0^s{\cdot}26 \qquad +\,1^s{\cdot}91 \qquad -3^s{\cdot}19$$

whence, by formula (7),—

$$a \textit{ (the initial rate)} = 2\left(a + \frac{b}{2}\right) - (a + b)$$

Also, by formula (8),—

$$b \text{ (the change of rate)} = 2\left(\overline{a+b} - \overline{a+\tfrac{b}{2}}\right)$$

And substituting numerical values, we shall find that for the several chronometers we have,—

	a	b
	s	s
Chron. Z	+ 0·18	+ 0·16
M	+ 1·83	+ 0·16
A	− 4·36	+ 2·34

With the values of a and b* thus determined, and substituted in formulæ (2) and (4), we proceed, as before, to compute the partial measurements, first from Hong Kong to Amoy, and secondly from Amoy to Shanghai.

First, by formula (2),—

$$M_1 = \lambda_1 - \left\{\lambda + \left(\tau a + \frac{\tau^2}{2t} b\right)\right\}$$

Here $\tau = 6$ and $\frac{\tau^2}{2t} = [-1{\cdot}954243]$.

And for the several chronometers we have,—

	τa	$\frac{\tau^2}{2t} b$	$\tau a + \frac{\tau^2}{2t} b$
	s	s	s
Chron. Z	+ 1·08	+ 0·14	+ 1·22
M	+ 10·98	+ 0·14	+ 11·12
A	− 26·16	+ 2·11	− 24·05

Whence, reducing the formula,—

	Z.	M.	A.
	h m s	h m s	h m s
Error of chron. at Hong Kong, Nov. 24d, or λ	−7 44 10·06	−7 59 43·36	−8 21 4·86
Corr. for rate $\left(\tau a + \frac{\tau^2}{2t} b\right)$	+1·22	+11·12	−24·05
Error on Nov. 30d, or $\lambda + \left(\tau a + \frac{\tau^2}{2t} b\right)$ ..	−7 44 8·84	−7 59 32·24	−8 21 28·91
Error at Amoy, or λ_1	−7 59 56·98	−8 15 23·85	−8 37 24·73
	−0 15 48·14	−0 15 51·61	−0 15 55·82

* As a check on the accuracy of calculation it will be observed, that the above values of a and b when combined, give values of $(a + b)$, agreeing with the previous statement of the rates at Shanghai, as given in the preceding page.

Hence, as the meridian distance, Hong Kong to Amoy, we have,—

		h	m	s
Chron. Z		-0	15	48·14
M				51·61
A				55·82
Mean ..		-0	15	51·86

Proceeding in a similar manner with the solution of formula (4),*

$$M_3 = \lambda' - \left\{\lambda_2 + \left(\overline{t - \tau'}\,a + \frac{\overline{t + \tau'} \, . \, \overline{t - \tau'}}{2\,t}\,b\right)\right\}$$

we should obtain, as the meridian distance from Amoy to Shanghai,

		h	m	s
Chron. Z		-0	13	28·91
M				25·52
A				21·37
Mean ..		-0	13	25·27

Collating this result with that of the previous partial measurement from Hong Kong to Amoy, it appears that their sum equals $-0^h\ 29^m\ 17^s{\cdot}13$; a quantity, as in the preceding example, almost exactly identical with the assumed difference of longitude of the terminal stations, adopted as the basis of the determination.

It will also be observed, that the individual results by each chronometer, taken separately, exhibit in a similar manner (as in the last example) the same degree of accordance; and also, if the several partial measurements by each chronometer, employing alternately the *initial* and *terminal* rates, are compared together, they will be found to have a very close agreement.†

Observation.—The method illustrated in the preceding examples of employing the known difference of longitude of the terminal stations to correct and adjust that of intermediate positions, seems capable of affording very useful results in conducting the work of surveys, since from its property of arranging its constituent parts in harmony with, and in subservience to, a

* In this case $\overline{t - \tau'} = 14$, and the factor $\frac{\overline{t + \tau'} \, . \, \overline{t - \tau'}}{2\,t} = [0{\cdot}959041]$.

† This example is taken from the observations made during the voyage of H.M.S. Sphinx, 1850–3.

primary and well-considered assumption, that the difference of longitude of the base stations may be taken of a given amount, it is susceptible of establishing among a mass of partial measurements a degree of accordance and consistency not otherwise attainable by any other mode of proceeding.

The differences of longitude of the base stations having then, in the first instance, been carefully deduced and definitively established, are subsequently admitted as data in the further prosecution of the work, and, with the aid of formulæ (5), (6), (7), and (8), made subservient to the ultimate systematic arrangement of the relative longitudes of intermediate positions.

We shall conclude these examples by giving one in illustration of the remarks made in Observation III. (chap. vii. p. 160), as to the occasional convenience of combining together in an aggregrate operation the performances of individual chronometers, so as to obtain the final mean result by one process. A meridian distance so determined may be looked on, as the result by a *fictitious* chronometer, representing the average indication of the several chronometers employed, and whose errors and rates respectively exhibit the mean arithmetic values of those actually appertaining to the chronometers under discussion.

Example. Suppose that the following observations were made by ten chronometers to determine the meridian distance between Bahia and Rio Janeiro:—

Errors and rates at Bahia (Fort San Pedro) at the mean epoch, May 10^{d}, 1836:—

	h	m	s	s
Z	+ 2	40	17·40	+ 1·60
A	+ 2	38	19·00	− 2·40
B	+ 3	1	18·60	− 1·75
C	+ 2	9	14·50	+ 8·33
D	+ 1	59	18·30	+ 17·20
E	+ 2	37	10·00	− 0·55
F	+ 2	0	11·70	+ 5·10
G	+ 1	37	24·20	+ 6·45
H	+ 4	9	37·30	− 24·30
I	+ 3	5	27·00	+ 3·10

After arrival at Rio Janeiro, by observations at Fort Villagagnan, the errors and rates of the chronometers at the mean epoch, May 31^d, were,—

		h	m	s	s
Z	+	2	59	26·30	+ 1·84
A	+	2	56	0·10	− 2·36
B	+	3	19	8·00	− 1·90
C	+	2	30	26·00	+ 8·05
D	+	2	24	1·50	+ 18·40
E	+	2	55	43·20	+ 1·10
F	+	2	20	44·30	+ 6·24
G	+	1	58	23·10	+ 6·10
H	+	4	19	57·40	− 22·20
I	+	3	25	56·00	+ 5·80

Here t, the interval between the epochs of the observations, is 21 days;

Also the mean value of a, the initial rate at Bahia, is $+1^s{\cdot}278$;

While the mean value of $(a+b)$, the terminal rate at Rio, is $+2^s{\cdot}107$;

Hence the *average* mean rate $\left(a+\frac{b}{2}\right)$, to be used in the computation, is $+1^s{\cdot}692$.

Likewise λ, the average error, or that appertaining to the *fictitious* chronometer at Bahia, is $2^h\ 35^m\ 49^s{\cdot}80$;

While λ', the corresponding error at Rio, is $2^h\ 54^m\ 58^s{\cdot}59$.

Then, by formula (1),

$$M = \lambda' - \left\{\lambda + t\left(a + \frac{b}{2}\right)\right\}$$

and solving numerically, we have,—

	h	m	s
Error of chron. at Bahia, May 10^d, or λ	+2	35	49·80
Correction for rate, or $t\left(a+\frac{b}{2}\right)$			+35·53
Error on May 31^d, or $\lambda + t\left(a+\frac{b}{2}\right)$ ..	+2	36	25·33
Error at Rio, or λ'	+2	54	58·59
M =	+0	18	33·26

Whence, for the meridian distance between Bahia and Rio by these ten chronometers, we have as above.

If we were to proceed separately for each individual chronometer, in accordance with the usual process, we should obtain results as follows:—

		h	m	s
Chron. Z	+	0	18	32·78
A				31·08
B				27·62
C				19·51
D				29·40
E				27·53
F				33·53
G				47·23
H				28·35
I				55·55
Mean ..	+	0	18	33·26

Hence it appears, that the result obtained from the aggregate process is identical with the mean result deduced in the usual manner, but by merging the individuality of performance of each chronometer in the general issue we are deprived of the power of detecting the eccentricity of any particular chronometer, or of analysing the results with a view to the adjustment of irregularities.

If, when a large number of chronometers is employed in any given measurement, it should appear on an inspection of the results that some of them exhibit considerable divergence from the general mean; and if it should further appear, on an examination of the records of the "Chronometer Journal," that the "second differences" of their daily comparisons indicated considerable instability of rate; then it might be proper for the computer to reject the evidence of such suspicious chronometers, and to decline to receive them into the general combination.

Now, in the example at present under discussion, chronometers C, G, and I, give values of the meridian distance differing considerably from those indicated by the other chronometers (their divergence from the general mean being respectively $-13^s{\cdot}75$, $+13^s{\cdot}97$, and $+22^s{\cdot}29$); if, then, a critical inspection of the "Chronometer Journal" afforded just reason for establishing against them the charge of irregularity of performance, the

computer would have been justified in disallowing their evidence, and in rejecting them from the final result.

Whence, in the present instance, the new mean result by the remaining seven chronometers would be,

$$+\ 0^{h}\quad 18^{m}\quad 30^{s}{\cdot}04$$

as the finally concluded meridian distance; but it is clear that the power of this critical correction is lost, if we only confine ourselves to an aggregate mode of solution, which mode of proceeding, therefore, however useful in other respects, should be restricted in practice to the purposes of duplicate reduction, as a check on the accuracy of computation.

The new mean result thus obtained is usually styled the *estimated mean*, in contradistinction to the *arithmetical mean* obtained in the first instance from the evidence of all the chronometers; the ultimate term, *corrected mean*, being held in reserve, to be finally appropriated by hydrographers to a result altered by subsequent or independent evidence.

CHAPTER IX.

ON THE DETERMINATION OF THE MERIDIAN DISTANCE BETWEEN TWO STATIONS, BY MEANS OF OBSERVATIONS, GIVING THE "TRAVELLING RATES" OF THE CHRONOMETERS EMPLOYED.

THE problem of determining the meridian distance between two places admits of a very neat and elegant solution, in cases where, after the observations for ascertaining the errors on time at the place, at the two stations, the chronometers are again brought back to the station from whence they originally started, and their errors again determined; it being premised that the whole of the observations on which the errors at the two places depend have been made within a reasonable interval.

The formulæ are thus explained,—

At a place, A, before starting on a voyage, let the error of a chronometer be α

After arrival at a station, B, let its error be β

Before leaving B, let its error be β'

And on return to the station, A, let its error be α'

Also, let n = the number of days between the first observation at A, and the first at B;

And let m = the number of days between the second observation at B, and the final one at A;

Then the quantity gained or lost by the chronometer, in the whole interval between the first and last observations at A, is obviously

$$\alpha' - \alpha$$

And during the detention at B,—

$$\beta' - \beta$$

consequently the gain or loss, during the double journey from A to B and back again, will be

$$\alpha' - \alpha - (\beta' - \beta)$$

and the *mean travelling rate* during the double journey is obviously

$$\frac{\alpha' - \alpha - (\beta' - \beta)}{m + n}$$

Consequently, the accumulation of the rate during the outward voyage from A to B will be

$$n\,\frac{\alpha' - \alpha - (\beta' - \beta)}{m + n}$$

and similarly for the return or homeward voyage from B back to A,—

$$m\,\frac{\alpha' - \alpha - (\beta' - \beta)}{m + n}$$

Hence, for the meridian distance from A to B, by the outward voyage, we have,—

$$M = \beta - (\alpha + k)$$

k being the correction for the accumulated rate, and substituting its value,

$$\begin{aligned} M &= \beta - \alpha - n\,\frac{\alpha' - \alpha - (\beta' - \beta)}{m + n} \\ &= \frac{m\,(\beta - \alpha) + n\,\beta - n\,\alpha - n\,\alpha' + n\,\alpha + n\,\beta' - n\,\beta}{m + n} \\ &= \frac{m\,(\beta - \alpha) + n\,(\beta' - \alpha')}{m + n} \qquad (1) \end{aligned}$$

If only one observation for the error were made at B, then β' and β are identical, and $\beta' - \beta = 0$; hence the formula becomes simplified, and we have

$$M = \beta - \alpha - \frac{n}{m + n}\,.\,(\alpha' - \alpha) \qquad (2)$$

Again, for the meridian distance by the homeward voyage from B to A,—

$$\begin{aligned} M &= \alpha' - (\beta' + k') \\ &= \alpha' - \beta' - m\,\frac{\alpha' - \alpha - (\beta' - \beta)}{m + n} \\ &= \frac{m\,\alpha' - m\,\beta' + n\,\alpha' - n\,\beta' - m\,\alpha' + m\,\alpha + m\,\beta' - m\,\beta}{m + n} \\ &= \frac{-\,n\,(\beta' - \alpha') - m\,(\beta - \alpha)}{m + n} \\ &= -\,\frac{m\,(\beta - \alpha) + n\,(\beta' - \alpha')}{m + n} \qquad (3) \end{aligned}$$

And again, as before, if only one observation were made at B, then

$$M = \alpha' - \beta - \frac{m}{m+n} \cdot (\alpha' - \alpha) \qquad (4)$$

Formulæ (1) and (3), it will be observed, give similar expressions for M, only with opposite signs, since obviously the meridian distances on the outward and homeward voyages are measured in opposite directions.

So also, again, if formulæ (2) and (4) are reduced to their simplest form, it will appear from (2) that

$$M = \frac{m(\beta - \alpha) + n(\beta - \alpha')}{m+n}$$

and from (4), that

$$M = -\frac{m(\beta - \alpha) + n(\beta - \alpha')}{m+n}$$

expressions giving equivalent values for M, but with opposite signs, and what formulæ (1) and (3) resolve themselves into, when, consistently with our hypothesis (only one observation having been made at B), $\beta' = \beta$.

For facility of computation, however, formulæ (2) and (4), as they stand, are to be preferred in practice in the actual reduction of observations.

These formulæ will be found very useful in two cases which may arise in practice: *first*, when it is wished to connect a maritime station with an inland position not accessible by the ship, or two inland stations with one another, when the operation can be performed by means of the available *pocket* chronometers; and *secondly*, when, in the course of any service, the ship leaving a given station, and calling at another, speedily returns to the first station again.

In both cases, the circumstance of the determination of the meridian distance being independent of the consideration of the *previous* or *subsequent* rates of the chronometers, and solely based on their travelling rates during the double journey, deduced from the observations for error at the two stations, gives it a great advantage, in point of elegance and conciseness, over the usual process.

Moreover, although an obvious remark, it is, perhaps, important to premise, that, in order that the observations should truly give the travelling rate unalloyed by any admix-

ture of that experienced by the chronometer when stationary, the observations for the error of the chronometer should be made as nearly as possible immediately before the period of departure from, and immediately after that of arrival at, the two stations; and, also, that the shorter the period elapsed during the double journey the better.

The necessity for these precautions, especially in the case of pocket-watches, whose rates when travelling, and thereby subjected to the influence of a land journey, may be very different from those they have when stationary, will, doubtless, on consideration, be quite apparent.

We shall now proceed to give some examples to illustrate the application of these formulæ.

Example 1. In the months of July and August, 1838, the following observations were made with three pocket chronometers, D, F, and P, to connect the observatories of Edinburgh and Greenwich:—

	D. Errors.	F. Errors.	P. Errors.
	h m s	h m s	h m s
July 11, 1 P.M. Grh.	−0 1 52·30	+0 0 9·80	−0 1 8·70
13, 8·30 P.M. Edh.	+0 10 39·60	+0 12 57·40	+0 11 32·30
Aug. 8, 1 P.M. Edh.	+0 9 44·10	+0 13 25·20	+0 11 16·00
10, noon, Grh.	−0 3 6·00	+0 0 48·20	−0 1 29·20

Here $\left.\begin{array}{l} n = 2^{d}\cdot322 \\ m = 1^{d}\cdot949 \end{array}\right\}$ allowing a correction of $0^{d}\cdot009$ for 13^{m}, the approximate difference of longitude.

$m + n = 4^{d}\cdot271$

Also in the formula to be employed, α and α' refer to the errors at Greenwich, and β and β' to those at Edinburgh.

By formula (1),—

$$M = \frac{m(\beta - \alpha) + n(\beta' - \alpha')}{m + n}$$

Likewise from the above observations we have,

	D.	F.	P.
	s	s	s
$\beta - \alpha =$	+ 751·90	+ 767·60	+ 761·00
$\beta' - \alpha' =$	+ 770·10	+ 757·00	+ 765·20
$m(\beta - \alpha)$	+1465·45	+1496·08	+1483·19
$n(\beta' - \alpha')$	+1788·17	+1757·75	+1776·92
Sum....	+3253·62	+3253·83	+3260·11
divided by $(m + n)$	+ 761·8	+ 761·8	+ 763·3

Hence,

$$M = +12^{m}\ 41^{s}{\cdot}80 \qquad +12^{m}\ 41^{s}{\cdot}80 \qquad +12^{m}\ 43^{s}{\cdot}30$$

Consequently the mean value of M, the meridian distance of Edinburgh from Greenwich, by these observations, is

$$+\ 0^{h}\ 12^{m}\ 42^{s}{\cdot}30$$

a result not differing much from the received longitude, $+\ 0^{h}\ 12^{m}\ 43^{s}{\cdot}60$ (as given in the "Nautical Almanac").*

* The formula given above,

$$M = \frac{m(\beta - \alpha) + n(\beta' - \alpha')}{m + n}$$

although expressed in its most symmetrical and elegant form, considered algebraically, and although conveniently adapted for general use when the numbers m and n are integral quantities, as they usually will be, and when consequently the calculation can be made by common arithmetic, is not so convenient as it might be, when the factors m and n are fractional numbers.

In this case it will be better to write

$$M = \frac{m}{m+n}(\beta - \alpha) + \frac{n}{m+n}(\beta' - \alpha')$$

and in computing from it, to find the logarithmic values of the factors,

$$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n}$$

In the case before us, the computation of the above example will then stand thus,

$$\frac{m}{m+n} = \frac{1{\cdot}949}{4{\cdot}271} = 0{\cdot}4563 = [-1{\cdot}659282]$$

$$\frac{n}{m+n} = \frac{2{\cdot}322}{4{\cdot}271} = 0{\cdot}5437 = [-1{\cdot}735332]$$

		D.	F.	P.
		s	s	s
Hence,	$\frac{m}{m+n}(\beta - \alpha) =$	343·1	350·3	347·3
And,	$\frac{n}{m+n}(\beta' - \alpha') =$	418·7	411·6	416·0
	Sum = M =	761·8	761·9	763·3

$$\text{Hence } M = +\ 12^{m}\ 41^{s}{\cdot}8 \qquad +\ 12^{m}\ 41^{s}{\cdot}9 \qquad +\ 12^{m}\ 43^{s}{\cdot}3$$

which agree with the previous calculation.

Of course the same reasoning applies to the reverse formula (3),

$$M = -\frac{m(\beta - \alpha) + n(\beta' - \alpha')}{m + n}$$

which may be written

$$M = -\left\{\frac{m}{m+n}(\beta - \alpha) + \frac{n}{m+n}(\beta' - \alpha')\right\}$$

Example 2. In July, 1845, the following observations were made to determine the meridian distance between Malacca Flagstaff and the Church at Sincapore:—

	Z. Errors. h m s	A. Errors. h m s	C. Errors. h m s
July $6^{d}\cdot0$ Sincapore	−7 44 15·42	−9 36 1·72	−4 7 6·92
$12^{d}\cdot5$ Malacca	−7 38 40·94	−9 29 38·54	−3 59 11·14
$16^{d}\cdot0$ Malacca	−7 39 8·70	−9 29 37·40	−3 58 22·70
$21^{d}\cdot0$ Sincapore	−7 46 12·68	−9 36 1·28	−4 3 40·08

Here, $n = 6\cdot5$

$m = 5\cdot0$

and $m + n = 11\cdot5$

The correction for the difference of longitude on the intervals being very small is disregarded.

Also in the formula, α and α' refer to the errors at Sincapore, and β and β' to those at Malacca.

By formula (3),—

$$M = -\frac{m(\beta - \alpha) + n(\beta' - \alpha')}{m + n}$$

And for the several chronometers we have, from the data above,—

	Z. s	A. s	C. s
$\beta - \alpha$	+ 334·48	+ 383·18	+ 475·78
$\beta' - \alpha'$	+ 423·98	+ 383·88	+ 317·38
$m(\beta - \alpha)$	+1672·40	+1915·90	+2378·90
$n(\beta' - \alpha')$	+2755·87	+2495·22	+2062·97
Sum....	+4428·27	+4411·12	+4441·87
divided by $(m + n)$	+ 385·06	+ 383·57	+ 386·25

Hence,

	h m s	h m s	h m s
M =	−0 6 25·06	−0 6 23·57	−0 6 26·25

And the mean value of M, the meridian distance of Sincapore east of Malacca, by the observations with these three chronometers, is,—

$$\overset{h}{-0}\ \overset{m}{6}\ \overset{s}{24\cdot96}$$

Remarks.—On examining the records from whence these observations are extracted, and on comparing the results obtained by our formula under discussion with those deduced in the ordinary manner,

separately, both by the outward and return voyages, we find as follows:—

Meridian Distance.

	Z.			A.			C.		
	h	m	s	h	m	s	h	m	s
Going	0	6	25·24	0	6	22·99	0	6	26·15
Returning..	0	6	24·93	0	6	24.03	0	6	26·33
Mean ..	0	6	25·08	0	6	23·51	0	6	26·24

Whence it appears that, in all cases, the above formula gives values of the meridian distance intermediate between those found by the ordinary process, and in every instance closely agreeing with their mean value. The same fact is also observable with reference to the results by several other chronometers employed on the same occasion.

We shall conclude with an example in which only one observation for the error of the chronometers was made at the second station.

Example 3. In June and July, 1851, observations for the meridian distance between Trincomalee (the Dockyard Flagstaff) and Madras Observatory* were made as follows, with three chronometers, Y, M, and R:—

		Y. Errors.			M. Errors.			R. Errors.		
d		h	m	s	h	m	s	h	m	s
June 23·00	Trincom.	−5	39	52·82	−5	45	28·82	−0	24	11·32
29·33	Madras	−5	36	39·30	−5	41	40·30	−0	24	39·30
July 14·00	Trincom.	−5	42	15·91	−5	45	54·93	−0	38	58·93

Here, $n = 6·33$

$m = 14·67$

And $$\frac{n}{m+n} = \frac{6·33}{21} = [-1·479185]$$

The correction for the difference of longitude on the intervals being very minute may be disregarded.

Also in the formula, α and α' refer to the errors at Trincomalee, while those at Madras are represented by β.

And by formula (2),—

$$M = \beta - \alpha - \frac{n}{m+n}(\alpha' - \alpha)$$

* The errors on mean time at the Observatory at Madras were obtained by noting the time of the flash of the evening gun at 8 P.M.; the exact Observatory mean time of which occurrence is duly published at Madras, in the Government Gazette: hence affording to ships in the roads a ready means for ascertaining the errors of their chronometers.

Now, for the several chronometers employed, we have from the above data,—

	Y.	M.	R.
	h m s	h m s	h m s
$\beta-\alpha$	+ 0 3 13·52	+ 0 3 48·52	− 0 0 27·98
$\alpha'-\alpha$	−143·09	−26·11	−887·61

	Y.	M.	R.
	h m s	h m s	h m s
$\beta-\alpha$	+ 0 3 13·52	+ 0 3 48·52	− 0 0 27·98
$\frac{n}{m+n}(\alpha'-\alpha)$	− 0 0 43·13	− 0 0 7·87	− 0 4 27·55
M	+ 0 3 56·65	+ 0 3 56·39	+ 0 3 59·57

Hence, by these observations, the mean value of M, the meridian distance of Madras Observatory, west of the Dockyard Flagstaff at Trincomalee, is,—

$$\overset{\text{h}}{+0}\ \overset{\text{m}}{3}\ \overset{\text{s}}{57{\cdot}54}$$

Again, reducing the meridian distance by formula (4),—

$$M = \alpha' - \beta - \frac{m}{m+n}(\alpha'-\alpha)$$

Now in this case the factor $\frac{m}{m+n} = \frac{14{\cdot}67}{21} = [-1{\cdot}844211]$.

Also for the several chronometers we have from the observed errors,—

	Y.	M.	R.
	h m s	h m s	h m s
$\alpha'-\beta$	− 0 5 36·61	− 0 4 14·63	− 0 14 19·63
$\frac{m}{m+n}(\alpha'-\alpha)$	− 0 1 39·96	− 0 0 18·24	− 0 10 20·06
M	− 0 3 56·65	− 0 3 56·39	− 0 3 59·57

Hence the mean value of M, as before, is,—

$$\overset{\text{h}}{-0}\ \overset{\text{m}}{3}\ \overset{\text{s}}{57{\cdot}54}$$

Representing the meridian distance of Trincomalee, east of Madras.

From which it appears, as we before remarked, that formulæ (2) and (4) give the same results, but with opposite signs; the one giving the meridian distance by the outward, and the other by the homeward, voyage.

CHAPTER X.

ON THE MODE OF RECORDING THE RESULTS OF CHRONOMETRIC MEASUREMENTS.

Having in the preceding chapters explained at some length all the points necessary to be attended to relating to the general management of chronometers, and having carefully led our readers onwards through all the processes, subordinate, in the first instance, to the determination of errors and rates, and subsequently conducive to the systematic deduction of "meridian distances," little now remains for us to add, beyond pointing out the details to be attended to in recording the results,—a matter of more importance than may at first sight appear, since the absence of minute details, or a loose habit of recording them, materially impairs the utility of chronometric measurements, and diminishes the ability of hydrographers to assign to them their just value, as compared with the previous determinations of others. Unnecessary difficulties are thus interposed to their comparison and incorporation with the labours of previous navigators, and hydrographic science in consequence derives but scanty benefit from, it may be, well-intentioned and often laborious exertions.

This part of our subject has already been so fully treated by Raper,* that we can scarcely do better than follow the footsteps of that able writer, having but little to add to the precepts enjoined by him.

I. In all reports of the results of chronometric measurements, the details should be preceded by a preliminary notation, descriptive of the several chronometers employed, embracing the following details:—the maker's name and number† — the chronometer's

* "Naut. Mag.," vol. vi. for 1839, p. 402; and "Practice of Navigation," p. 281.

† The makers' names and numbers should be recorded, in justice to the reputation of the makers themselves, by facilitating the publication of the performances of first-rate chronometers; and also, on the other hand, the failure of indifferent ones: facts which, without this precaution, are masked by the uncertainty of the

distinctive letter*—whether box or pocket—whether one, two, or eight-day—where stowed in the ship, and how—position of marks on dial-plate as to fore-and-aft line—time of winding and comparing, &c. &c. As the greater part of these details are permanent facts, incapable of, or not liable to, alteration, they should be fully recorded once for all. If any subsequent alterations take place, if accident befall any of the chronometers, or if new ones are received, &c., special notation of the fact should subsequently be inserted in the reports from time to time, in chronological order.

It may also be proper to place on record, and explain any special symbols or abbreviations† which may be adopted in the subsequent pages; and a brief reference to the modes of observation and instruments employed, and to the formulæ of computation adopted in the reduction of the observations, if characterised by anything special, may also be given with propriety.

II. It is advisable that the records of the performances of the chronometers should be specified in chronological order, and that the several measurements be numbered consecutively. "This succession," as Raper observes, "admits of specifying in its proper place any events which may relate to the chronometers collectively or individually—as, for example, running down, accidents, exchange, or receiving new ones, &c. The chronological order, also, is evidently the only one in which we may expect to find that connexion which is absolutely necessary in considering a series of chronometric determinations, or to search with success for the origin or first appearance of an observed discrepancy among various results."

literal symbols, which, however useful to the persons charged with the superintendence of their performances, are still only to be regarded as private distinguishing marks, and therefore unsuited for ultimate official publication, without accompanying explanation. Since, moreover, the same chronometer may be employed at different times, on different voyages, and as a knowledge of its history, as Raper remarks, influences the value of its testimony, its description by its permanent characteristics seems desirable, since the distinctive letters assigned to it on different voyages may, of course, be different, and do not necessarily convey any idea of connexion with former periods.

* A, B, or C, &c. See *ante*, p. 26.

† Raper suggests *ch* for chronometer; *d*, for days; D L, for difference of longitude; and that the extreme difference of results should be denoted by the number of seconds enclosed in brackets, implying limit or boundary; thus, [7^s]. As these symbols are simple and self-suggestive, they may with propriety be recommended for adoption.

In cases where, in any series of measurements, the ship returns to the same ground, and repeats the determination of any given chronometric link, a special notation may be made, and attention called to the fact, so that a comparison of the values of the successive results may be instituted with facility and convenience.

III. It is absolutely necessary to specify or to describe the *exact spot of observation* at the places visited. For instance, it would be no use to report that the meridian distance by so many chronometers between Plymouth and Lisbon was "so-and-so," because Plymouth and Lisbon, being both large places, covering some miles of ground, such information respecting them is defective in precision, and however valuable they might be in other respects, such loosely-recorded facts must needs be disregarded.

It is advisable, therefore, that the assumed latitude and longitude of the place of observation (with the authority for them when necessary) be specified, and its topographical position accurately described, in accordance with the precepts laid down on this matter in a previous chapter (see *ante*, p. 60); and this should invariably be done in recording observations made at new stations.

IV. The modes by which the observations for time, on which the errors and rates employed in the computations depend, should also be stated, whether equal altitudes, single altitudes, or otherwise. Also the number of days at each place occupied in determining the rates,* and likewise the intervals between the several epochs of the observations. If any of the observations have been made with the sea horizon, the circumstance should be specially noted.

V. When several chronometers are employed, the results by each chronometer should be exhibited. The general *arithmetic mean* should be given, and also the *estimated mean*, obtained

* Specific particulars respecting the rates are, it would appear, of only secondary consequence to the hydrographer, as he can never be in a situation to employ this information to such advantage as the observer himself. Proper details relative to the character of the rate, such as how determined, and whether steady or unsteady, should, however, accompany every chronometric determination; and as we have before observed (p. 35), the original books containing all the details on these matters should be neatly and methodically kept, so as to be available for inspection and reference when needed.

by rejecting the results of those chronometers which are discordant or irregular, and merely retaining those whose performances during the voyage seem unexceptionable; or even, in some cases, in accordance with the practice of some computers, by giving more or less weight to the several results, according to the performance of each chronometer, and of which the observer alone can be a judge.

There is, perhaps, no question relating to the deduction of meridian distances, which involves considerations of greater delicacy and importance, or which requires to be approached with more caution than that relating to the establishment of an *estimated mean.*

The computer is so liable to have his judgment warped by predilections in favour of some preconceived and desired solution, and even, possibly, by his partiality for particular chronometers, that there is a constant temptation to be either too hasty in rejecting particular results because they do not accord either with the general mean or with previous expectations, or to be unduly desirous of retaining them because they happen to agree with those of others, or to harmonise with his previous ideas.

We are by no means in favour of the practice, which has been sometimes adopted, of assigning weights to the values of particular results, dependent on the assumed character of the chronometers employed; such a practice affords an opening to the possibility of all sorts of "cooking" and "trimming," since sets of observations may thus be twisted to support any required determination, although certainly at the expense of their integrity.

Even the practice of altogether rejecting the results by particular chronometers, because they exhibit a divergence from those of others, is not unfraught with danger, and should not be adopted without discreet and critical consideration. The mere divergence of a given result from the general mean, unless excessive in amount, would not in itself seem to justify its rejection, unless at the same time an examination of the records of the "daily comparisons" exhibited also unmistakable indications of instability of rate; and this consideration is of the greater weight when measurements occupying long periods of time are under discussion, since a degree of apparent irregularity, which might justly authorise the rejection of a particular result in a short run occupying but a few days, would be insufficient to

sanction that proceeding if a considerable interval elapsed during the voyage.

It is, perhaps, impossible to lay down any specific rules on this subject for the guidance of computers; all that we can do is to enjoin caution and discreetness, and to indicate, as above, some general considerations to aid the judgment.*

The final results, both of the *arithmetic* and *estimated means*, should be given in *arc* as well as in *time*. In *time*, because it is in that form that they first develop themselves under the hand of the computer, and are most convenient for inter-comparison; and in *arc*, for the convenience of geographical purposes, because the unit of measure in navigation being a mile or minute, and charts having a scale divided accordingly, the diff. long. in arc is absolutely necessary in comparing two charts.

VI. The extreme difference of the greatest and least results

* It might, perhaps, assist to aid our judgment in estimating the values of chronometric determinations, and possibly to check the tendency to a too hasty rejection of particular results which swerve from the general mean, or do not tally with preconceived ideas, if we endeavour to institute a comparison between the accordance of the results obtained from individual chronometers, in any given measurements, and those which are found to exist between the single results of various kinds of astronomical observations.

Observations of solar eclipses, occultations of stars by the moon, moon-culminating stars; eclipses of Jupiter's satellites and lunar distances, have all been employed, at various times, by different observers, as astronomical data for the definitive settlement of the absolute longitude of many fundamental stations. It may be instructive to make a cursory examination of the degree of accordance which is found to prevail among observations of those kinds, and it seems not unfair to contrast results by individual chronometers with the issue of single astronomical observations; and if we mistake not, the tendency of such a comparison would be to reconcile observers to a much greater amount of discrepancy than they are usually inclined to tolerate.

Accurate observations of solar eclipses and occultations of stars by the moon, especially when central, and observed under favourable circumstances, have been considered to yield very reliable results for the longitudes of the places of observation. Yet among the results recorded by M. Daussy, in the "Connaissance des Tems," and collected by Raper in his paper on Maritime Positions, in the "Nautical Magazine," we find the differences between individual results ranging as high as 31^s for eclipses, and 56^s for occultations.

The results by the eclipses of Jupiter's satellites are greatly influenced by the power of the telescope employed, and the discrepancies between single observations consequently range much higher. Raper records an instance of four eclipses of the satellites observed by the late Sir E. Home, at Port Royal, with an excellent telescope, in which the range between the extreme observations amounted to 214^s. This,

by the different chronometers employed, or the *range*, as it is sometimes called, should be stated, to facilitate the estimation of the general dependence to be placed on each determination, by showing whether the chronometers went well together or not; for though their going together does not prove that all or any of them are right, their not going together proves that some of them are wrong.

VII. Every result should be given, without any regard to *whether it agrees or not with received determinations.* Many received positions, as Raper observes, are very erroneous; and the only means by which they can be decisively rectified are the comparisons of independent and impartial evidence.

VIII. Since it is known that changes of temperature influence the performances of chronometers, the range of the temperature of the chronometer-room should be stated, with accompanying remarks, if its fluctuations had been excessive.

doubtless, was a very extreme case, but discrepancies of as much as 30^s or 40^s seem not uncommon.

Observations of moon-culminating stars have been greatly commended as a means of determining differences of longitude, and much has been expected from them. In practice, however, they have scarcely maintained their reputation. In a large mass of observations recorded in the "Memoirs of the Astronomical Society," connecting the Observatory at Madras with Greenwich, Cambridge, and Edinburgh, the discrepancies between extreme individual results range as high as $32^s \cdot 5$; and in a similar series of corresponding observations, connecting the observatories at Madras and the Cape, the range of difference amounts to $31^s \cdot 8$; and in those connecting Bombay with Greenwich and Bushey, to $30^s \cdot 4$ and $33^s \cdot 9$ respectively.

The results by Lunar Distances are, it is well known, still more uncertain and unsatisfactory, not only as regards isolated determinations, but often even as regards the mean of masses of observations; so much so, that the judgment of modern hydrographers seems inclined to reject the lunar method altogether as a means of definitively establishing fundamental positions. (See note, *ante*, p. 4.)

If, then, we find such serious discrepancies to prevail between the individual results of astronomical observations, often obtained by skilful observers in established observatories, and with first-rate instruments, surely we ought to view with patience, and be inclined to reject with caution, discrepancies existing among the results of individual chronometers, which, however perfectly constructed, are but machines after all.

It is, of course, impossible, in the brief limits of a note, to attempt to do justice to this interesting inquiry, or to do more than briefly suggest a few heads for consideration, and indicate to the studious reader the sources from which further information can be obtained; and with these remarks we commend the subject to the critical consideration of the scientific student.— (*See "Memoirs Ast. Soc.,"* vol. iii. p. 369; vol. xii. pp. 119 and 133; "*Connaissance des Tems,*" vol. 1835–6, additions; "*Nautical Magazine,*" *Raper on Longitudes*, vol. 1839, &c.)

Uniformity of temperature would justify the inference of uniformity of rate, while violent changes would give rise to a suspicion of irregularity.

Also, since there is reason to suspect that the influence of the ship's magnetism may in some cases produce an appreciable effect on the chronometers, it is proper to indicate the general direction of the ship's head during the voyage. Precision in the indication would, indeed, be useless; but a knowledge of this element might, perhaps, be of service on some occasions. In steam-ships, where the introduction of the boilers and machinery concentrates large masses of iron in the interior of the ship, notations on this subject would seem to be important, and may be highly useful, since it is only by the careful record of observed facts that we can ever hope to penetrate the obscurity which at present attends the subject of magnetic influence as affecting chronometers.

In accordance with the principles laid down in the preceding remarks, the records of a meridian distance would be exhibited in the following manner:—

Example 1.

No. (   )   H.M.S. __________ Capt. N.   May, 1836.

Bahia (*Fort San Pedro*) to Rio Janeiro (*Fort Villagagnan*).

		h	m	s	
Chron.	Z	+ 0	18	32·78	
	A			31·08	
	B			27·62	Temp. 80° to 74°.
	C			19·51	
	D			29·40	Ship's head S.S.W.
	E			27·53	
	F			33·53	Rates at both places by
	G			47·23	Eq. Alt. Intervals 7
	H			28·35	and 5 days.
	I			55·55	

Arithmetic mean, + $0^h\ 18^m\ 33^s{\cdot}26$, or $4^\circ\ 38'\ 19''$; Estimated mean, $0^h\ 10^m\ 30^s{\cdot}04$, or $4^\circ\ 37'\ 30''{\cdot}6$ (rejecting C, G, and I, whose rates were unsteady); 7 ch, 21^d, [6^s].

Example 2.

No. (   ) H.M.S.__________ Capt. N. May, 1844.

Cape Upstart to Sir Charles Hardy's Islands (N.E. Coast of Australia).

Chron.	h m s
Z	+ 0 17 9·12
A	8·09
B	15·16
C	3·56
D	16·05
E	9·43
F	6·18
G	7·63
H	14·11
I	10·23
K	5·51
L	3·83

Spot of observation, Cape Upstart (a small valley, near a white rock off the west point of the Cape).
Lat. 19° 43′ 0″ S.
Long. 147 48 0 E.

Spot of observation, Sir C. Hardy's Islands (a sandy beach west side of the centre island).
Lat. 11° 55′ 15″ S.
Long. 143 31 0 E.

Temp. 76° to 82°.

Ship's head N.N.W.

Errors and rates at both places by Eq. Alt. Intervals, 4 days.

Arithmetic mean, $+0^h\ 17^m\ 9^s{\cdot}07$; Estimated mean, $+0^h\ 17^m\ 9^s{\cdot}10$, or $4°\ 17'\ 16''{\cdot}5$ (rejecting H and L, whose rates were unsteady), 10 ch, 19^d, [$12^s{\cdot}49$].

No. (   ) Cape Upstart to Lizard Island. May, 1844.

Chron.	h m s
Z	+ 0 9 12·50
A	11·31
B	16·69
C	10·40
D	17·10
E	11·73
F	8·10
G	9·45
H	15·00
I	13·70
K	7·16
L	6·00

Spot of observation, Lizard Island (a small sandy bay, N.W. side of Island).
Lat. 14° 40′ 0″ S.
Long. 145 31 0 E.

Temp. 76° to 80°.

Ship's head N.N.W.

Errors on time only obtained at Lizard Island by Eq. Alt. Former rates employed, as in preceding measurement.

Arithmetic mean, $+0^h\ 9^m\ 11^s{\cdot}59$; Estimated mean, $+0^h\ 9^m\ 11^s{\cdot}81$, or $2°\ 17'\ 57''$ (rejecting H and L as before), 10 ch, 8^d, [$9^s{\cdot}94$].

No. ( ) Lizard Island to Sir Charles Hardy's Islands. May, 1844.

		h	m	s	
Chron. Z	+	0	7	57·50	
A				56·57	
B				57·92	Temp. 80° to 82°.
C				53·03	
D				59·10	Ship's head N.N.W.
E				58·00	
F				58·00	Error on time only found at
G				57·20	Lizard Island by Eq. Alt.
H				59·50	Former rates employed as in
I				56·00	preceding measurements.
K				58·60	
L				56·40	

Arithmetic mean, $+ 0^h 7^m 57^s{\cdot}32$; Estimated mean, $+ 0^h 7^m 57^s{\cdot}19$, or 1° 59′ 18″ (rejecting H and L as before), 10 ch, 11^d, [$6^s{\cdot}07$].

Example 3.

No. ( ) H.M.S. ________ Capt. N. Dec. 1850. Jan. 1851.

Shanghai to Hong Kong.

		h	m	s	
Chron. Z	+	0	29	15·42	Spot of observation, Shanghai, Mr. Beale's house.
R				12·00	Lat. 31° 15′ 15″ N. Long. 121 29 0 E.
					Spot of observation, Hong Kong, Dent's Wharf.
Y				15·60	Lat. 22° 16′ 27″ N. Long. 114 10 0 E.
M				24·50	Temp. 43° to 57°. Ship's head S.S.W.
A				18·95	Errors and rates at both places by Eq. Alt. Intervals, 8 and 19 days.

Arithmetic mean, which we adopt, $+ 0^h\ 29^m\ 17^s{\cdot}29$. Beale's house is $0^s{\cdot}56$ East of the Consular Flagstaff at Shanghai, and Dent's Wharf $0^s{\cdot}26$ West of Victoria Cathedral at Hong Kong. Hence we have for the corrected meridian distance between the Flagstaff and the Cathedral, $0^h\ 29^m\ 16^s{\cdot}47$, or 7° 19′ 7″, 5 ch, $22^d{\cdot}5$, [$12^s{\cdot}50$].

In the preceding examples, taken from actual observations made on board H. M. ships, we have not been able to follow out the precepts laid down and insisted on so strongly in these pages, viz. to record in all cases the mean daily temperature of the chronometers at the time of winding, as indicated by the read-

ings of the maximum and minimum thermometers. The actual records of these observations have not given the mean daily temperatures with sufficient precision to enable us to do so. In Form No. III. (Appendix, p. 222), which is an amplification of the above mode of record, in a ruled form, provision has been made in the columns for the future careful notation of this highly important element.

In cases where it may not be deemed necessary to record the results of the chronometric measurements with the same minuteness of detail as is exhibited in the above examples, a more succinct method may be adopted; and a tabular form adapted to that purpose will be found in the Appendix, p. 223, in which the results may be briefly exhibited in a more popular shape.

APPENDIX.

TABLE

For converting Intervals of Time, or Longitude, into Decimals of a Day.

Long.	Time.	Decimals of a Day.	Long.	Time.	Decimals of a Day.	Long.	Time.	Decimals of a Day.
°	h		° ′	m		° ′	m	
15	1	·0417	0 15	1	·0007	7 45	31	·0215
30	2	·0833	0 30	2	·0014	8 0	32	·0222
45	3	·1250	0 45	3	·0021	8 15	33	·0229
60	4	·1667	1 0	4	·0028	8 30	34	·0236
75	5	·2083	1 15	5	·0035	8 45	35	·0243
90	6	·2500	1 30	6	·0042	9 0	36	·0250
105	7	·2917	1 45	7	·0049	9 15	37	·0257
120	8	·3333	2 0	8	·0056	9 30	38	·0264
135	9	·3750	2 15	9	·0062	9 45	39	·0271
150	10	·4167	2 30	10	·0069	10 0	40	·0278
165	11	·4583	2 45	11	·0076	10 15	41	·0285
180	12	·5000	3 0	12	·0083	10 30	42	·0292
195	13	·5417	3 15	13	·0090	10 45	43	·0299
210	14	·5833	3 30	14	·0097	11 0	44	·0306
225	15	·6250	3 45	15	·0104	11 15	45	·0312
240	16	·6667	4 0	16	·0111	11 30	46	·0319
255	17	·7083	4 15	17	·0118	11 45	47	·0326
270	18	·7500	4 30	18	·0125	12 0	48	·0333
285	19	·7917	4 45	19	·0132	12 15	49	·0340
300	20	·8333	5 0	20	·0139	12 30	50	·0347
315	21	·8750	5 15	21	·0146	12 45	51	·0354
330	22	·9167	5 30	22	·0153	13 0	52	·0361
345	23	·9583	5 45	23	·0160	13 15	53	·0368
360	24	1·0000	6 0	24	·0167	13 30	54	·0375
			6 15	25	·0174	13 45	55	·0382
			6 30	26	·0181	14 0	56	·0389
			6 45	27	·0187	14 15	57	·0396
			7 0	28	·0194	14 30	58	·0403
			7 15	29	·0201	14 45	59	·0410
			7 30	30	·0208	15 0	60	·0417

Examples illustrating the Application of the preceding Table.

Example 1. Let it be required to express the interval between $9^h\ 17^m$ A.M., on Nov. 9th, and $8^h\ 46^m$ A.M., on Nov. 17th, in decimals of a day.

Here $21^h = 0{\cdot}8750$		Also, $20^h = 0{\cdot}8333$	
and $17^m = 0{\cdot}0118$		and $46^m = 0{\cdot}0319$	
Sum ..	$0{\cdot}8868$	Sum ..	$0{\cdot}8652$

Therefore the first date is Nov.		$8^d{\cdot}8868$
second	—	$16{\cdot}8652$
and hence the interval	=	$7{\cdot}9784$

Example 2. Required the interval between Apparent Noon on Feb. 20th, 1855, at Portsmouth, and March 19th, $3^h\ 11^m$ P.M., mean time at Bermuda.

Apparent Noon on Feb. 20th corresponds to $0^h\ 14^m$ mean time (the "equation of time" being 14^m to be added to apparent time), therefore the date at Portsmouth is, Feb. $20^d{\cdot}0097$.

Again, $3^h\ 11^m$ correspond to $0^d{\cdot}1326$, therefore the date at Bermuda is, March $19^d{\cdot}1326$, or Feb. $47^d{\cdot}1326$.

Hence the apparent interval between Feb. $20^d{\cdot}0097$ and Feb. $47^d{\cdot}1326$, is $27^d{\cdot}1229$; but the difference of longitude between Portsmouth and Bermuda is $63^\circ\ 46'$, the fraction corresponding to which is $0^d{\cdot}1771$, to be added to the apparent interval, because the ship loses time by sailing to the westward. Hence the true interval is $27^d{\cdot}300$.*

* See Observation II. p. 159.

SUMMARY OF INSTRUCTIONS FOR THE MANAGEMENT AND USE OF CHRONOMETERS.

The following summary of instructions for the management and use of chronometers has recently been prepared at the Admiralty, and forms part of the new code of instructions for the government of the fleet:—

I.— *On Embarking.*

1. Whenever chronometers are to be transported, clamp the catch of their gimbol rings and carry them by hand, or slung with a line, or in a handkerchief, taking care not to give them any shock or circular or oscillating motion.

2. When embarked, stow them in the case prepared for them on beds of horsehair, shreds of bunting, or raw cotton, padding them around and between with the same soft material, so as to prevent the possibility of motion or concussion. The line joining their XII. and VI. hour marks should be all in the same direction, and parallel to the keel. Never use sawdust or wood shavings.

3. Release the catch of the gimbol rings; see that the gimbols work freely, but not too easily.

The chronometers being once secured in their places are *on no account to be subsequently moved or displaced** until relanded at home.

4. The chronometer case should be covered over with a covering of coarse woollen cloth to guard them from sudden changes of temperature.

II.—*On Winding and Comparing.*

1. Wind up daily at the same hour (8 A.M.), counting the turns, and winding carefully until the key is felt to butt; eight-day chronometers on Sundays.

2. Immediately after winding, compare each of the chronometers with the "standard" (the best chronometer, previously selected,) and note the comparisons in the chronometer journal, Form No. 1.

3. Note also at the same time the thermometer (which should be kept within the chronometer box) and also the barometer.

* The time for all observations on shore or on board is to be taken, in the first instance, by a pocket chronometer or an assistant watch showing seconds, and the times then shown by each of the chronometers are to be obtained from it, by comparing it with them both before and after the observations. See Sec. II.

III.—*On Errors and Rates.*

1. Chronometers being virtually intended for determining the "difference of longitude" or "meridian distance" between places visited by the ship, as well as for her safe navigation, their errors and rates must be very carefully ascertained. The ERRORS are best found by "equal altitude" observations of the sun, or in default of them, by single altitudes A.M. or P.M.: all such observations should be made with an artificial horizon on shore.

2. The RATES are to be ascertained by comparing their errors on local mean time, obtained consecutively at convenient intervals of not less than *five* or more than *ten* days; *seven* days is a convenient average interval. The difference of the errors divided by the number of days elapsed between them gives the mean daily rate of the interval.

3. At places where time signals are established for giving daily "local or Greenwich mean time," they may be taken advantage of, and the errors and rates deduced from them: but independent astronomical observations are most to be preferred. The errors should be recorded, with the rates deduced from them, as shown in Form No. II.

IV.—*On Meridian Distances.*

1. The chronometric longitudes of places visited are not required, but the "meridian distance," or difference of longitude *in time*, as shown by each chronometer between those places from the observations,* is to be carefully recorded, as in Form No. III.†

V.—*On Position of Place of Observation.*

1. It is very important that the exact site of the observations should be distinctly specified. At a well-frequented place, it is advisable that they be made on the site previously selected by former observers, as great confusion arises from the unnecessary multiplication of sites of observation.

* The value of a meridian distance depends mainly on the care with which the errors and rates have been determined at each of the terminal stations, and the errors at intermediate ones; on the shortness of the run between the places called at, and on the number of chronometers employed.

† It is extremely desirable that a record of the mean daily temperature of the chronometer case, extracted from the chronometer journal, should always accompany the returns of meridian distances, forwarded at any time to the Admiralty. In the present state of chronometric science the investigation of the effects of temperature is a question of much interest, and can only be successfully accomplished by aid of the records of careful observations.

2. If it be a new station, its situation should be carefully described, its latitude and approximate longitude be given, or its connexion with some adjacent known position.

VI.—*The Chronometer Journal.*

1. The chronometer journal, kept in Form No. I., is to contain the record of the daily comparisons at the time of winding.

2. Notice should be taken in the column of remarks of any circumstances likely to affect the performance of the chronometers, such as gales of wind, violent motion of the ship, her striking the ground, heavy firing of guns in action or for exercise, accidents to or stoppage or removal of any of the chronometers, storms of thunder, lightning, and general direction of the ship's head, &c.

3. The daily rates of the chronometers, determined from time to time by observation, should also be noted as a record, and for the purpose of occasional comparison with the column of second differences.

4. The chronometer journal, being *neatly* and *methodically* kept in the manner above described, there will be no necessity for keeping a fair copy of it, the *original* being carefully preserved in readiness for transmission to the Admiralty, if required.

VII.—*Official Returns.*

1. As the results arising from the good management of chronometers are only permanently required for the purpose of settling longitudes, Form No. III., when filled up, is to be transmitted annually to the Secretary of the Admiralty, with the ship's remark book. But as there is much probability of the others, No. I. and No. II., being also required, officers are strictly enjoined to pay especial attention to all the foregoing particulars, both as to the precepts they contain and the forms directed by them to be kept, in order that when these are applied for, on any occasion, for the investigation of results in longitude, they may be immediately forthcoming, in a satisfactory condition, for reference in the Hydrographic Office.

On the ship being paid off the chronometer journal is to be forwarded, with the other returns, to the Secretary of the Admiralty.

Form No. 1.—Chronometer Journal for the record of Daily Comparisons of Chronometers.

H.M.S. ____________

Date.	Bar. and Ther.	Max. and Min. Ther.	Chron. A.	2d Diff.	Chron. B.	2d Diff.	Chron. C.	2d Diff.	Initials of Comparers.	Remarks.
1860. Sun. Jan. 1.	in. 29·91 76°	80° 78	h m s Z 2 15 0 10 50 7 3 24 53		h m s 2 15 30 7 20 1 6 55 29		h m s 2 16 0 1 11 54 1 4 6		C.S. W.B.	Heavy Gale. Much Motion.
Mon. 2.	in. 30·32 69°	78 71	Z 2 39 0 11 14 5·3 3 24 54·7	s 1·7	2 39 30 7 43 48·8 6 55 41·2	s 12·2	2 40 0 1 35 55·5 1 4 4·5	s 1·5	C.S. G.C.	Confused Cross Sea.
Tues. 3.	in. 30·41 71°	74 66	Z 2 24 0 10 59 5 3 24 55	0·3	2 24 30 7 28 58 6 55 32	10·8	2 25 0 1 20 56·3 1 4 3·7	0·8	G.C. A.M.	Less Motion.
Wed. 4.	in. 30·34 72°	76 70	Z 2 27 0 11 2 4·2 3 24 55·8	0·8	2 27 30 7 31 28 6 56 2	10·0	2 28 0 1 23 57·3 1 4 2·7	1·0	W.B. A.M.	Exercised firing at Night Quarters.
Thur. 5.										
Fri. 6.										
Sat. 7.										
Sun. 8.										

Form No. 1 (A)*.—Chronometer Journal for fair copy of Comparisons, if thought necessary.

H.M.S.________________

Date.	Z — A	2d Diff.	Z — B	2d Diff.	Z — C	2d Diff.	Bar. and Mean Temp.	Remarks.
1860.								
Jan.	h m s	s	h m s	s	h m s	s	in.	
Sun. 1	3 24 53		6 55 29		1 4 6		29·91 79°	Heavy Gale. Much Motion.
Mon. 2	3 24 54·7	1·7	6 55 41·2	12·2	1 4 4·5	1·5	30·32 74°	Confused Cross Sea.
Tues. 3	3 24 55	0·3	6 55 52	10·8	1 4 3·7	0·8	30·41 70°	Less Motion.
Wed. 4	3 24 55·8	0·8	6 56 2	10·0	1 4 2·7	1·0	30·34 73°	Exercised firing at Night Quarters.
Thur. 5	3 24 56·5	0·7	6 56 13	11·0	1 4 1·5	1·2	30·19 72°	4th. 8 A.M. arrived at Sincapore.
Fri. 6	3 24 57·3	0·8	6 56 23·5	10·5	1 4 0·5	1·0	30·24 72°	Errors and Rates at the Mean Epoch:—
Sat. 7	3 24 58	0·7	6 56 34	10·5	1 3 59·7	0·8	30·16 70°	Jan. 7d·139, by single-alt. P.M.
Sun. 8	3 24 59	1·0	6 56 45	11·0	1 3 58·5	1·2	30·16 72°	obs. on the 4th and 10th.
Mon. 9	3 24 59·5	0·5	6 56 55·5	10·5	1 3 57·5	1·0	30·10 71°	h m s s Z—5 35 10·0 +2·10 A—9 0 8·3 +1·35
Tues. 10	3 25 0·5	1·0	6 57 6·7	11·2	1 3 56·7	0·8	30·14 73°	B—0 31 47·0 −8·50 C—6 39 9·5 +3·15
Wed. 11	3 25 1·5	1·0	6 57 18·0	11·3	1 3 56·0	0·7	30·16 72°	
Thur. 12								
Fri. 13								
Sat. 14								

* This is the same form as Form No. 2, *ante*, p. 29; we here call it No. 1 (A), so as not to interfere with the numbering of the forms adopted by the Admiralty.

Remarks on the Forms Nos. I. and No. I. (A.) *for Chronometer Journal.*

The Chronometer Journal, *Form No. I.*, should be kept ruled up for use in a book of the size of foolscap paper; each page may be ruled to hold ten days' comparisons, three pages thus sufficing for a month. The column for remarks should be of ample width, and if necessary, the form should be carried across the page, so as to accommodate any required number of chronometers.

Form No. I. (A.) may be ruled so as to exhibit a month's comparisons in each page; and when the number of chronometers renders it necessary, the ruling may be carried across into the opposite page, as before.

It will be useful to note from time to time, in the column of remarks, the errors and rates of the chronometers as determined by observation. In Form No. I. (A), if a fair copy is kept; if not, in Form No. I.

If preferred for neatness, the months may be described by the Roman numerals I. to XII., and the days of the week by their symbols, as follows:—

☉ Sunday.

☽ Monday.	♃ Thursday.
♂ Tuesday.	♀ Friday.
☿ Wednesday.	♄ Saturday.

In Form No. I. (A.) the mean temperature is the mean of the readings of the maximum and minimum thermometer as noted at the time of winding; also the column containing the initials of the comparers is omitted, as a point of no permanent importance.

As a general rule it is not recommended to keep any copy of the comparisons. The *original* book in Form No. I., *neatly* and *methodically* kept, will be sufficient. If, however, on any specific occasion, copy of the comparisons should be wanted, for the period embracing any particular measurement, Form No. I. (A.) is conveniently available for that purpose.

No.____ *Form No. II.* Returns to be numbered consecutively; month and year to be specified.

18___. Return of observed Errors and Rates of Chronometers.

H.M.S.__________ A. B., Esq., Captain, __________ Station.

Reference Letter.	Errors of Chronometer on Local Mean Time. By (1) Date and Time (2)	Errors of Chronometer on Local Mean Time. By (1) Date and Time (2)	At the Mean Epoch (3).		Mean Daily Temperature during Period of Rating.	Place and Spot of Observation.	Remarks.
			Concluded Mean Error (4).	Concluded Mean Rate (5).			
Z. A. B. C. D.					(6.) Here insert the mean of the two thermometer readings at time of winding for every day between the periods of observation for errors.	(7.) Here distinctly specify the place and spot of observation. If a new station, give its latitude and approximate longitude, or its connexion with some known point.	(8.) Here insert anything noteworthy, likely to affect the chronometers or the value of the observations; weather on days of observation, &c. &c.
Z. A. B. C. D.	(1.) By (2.) Date and Time. N.B.—If errors alone have been ascertained, they are to be recorded in this column.	(1.) By (2.) Date and Time.					

Approved, A. B., Captain. C. D., Master.

(1.) Here insert nature of observation: Equal altitudes, single altitudes, A.M. or P.M., &c. &c. &c.

(2.) Here insert date and time. It will be convenient to express the latter decimally: thus, June 9th, 8·40 A.M. would be written June $8^{d}\cdot861$; May 21st, 3 P.M., May $21^{d}\cdot125$. See Appendix, "Notes on Chronometers," Table for converting Intervals of Time into Decimals of a Day, p. 213.

(3.) Here insert the mean date and time on which the two errors depend: thus, if the first error belonged to Jan. $5^{d}\cdot880$, and the second to Jan. $12^{d}\cdot872$, the mean epoch would be Jan. $9^{d}\cdot376$.

(4.) Here insert the mean of the two observed errors.

(5.) Here insert the difference of the two errors, divided by the interval in days elapsed.

No.___ *Form No. III.* Returns to be numbered consecutively, and month and year specified.

Meridian distance ____________ to ____________

Measured in H.M.S. ____________ A. B., Esq., Captain,

between the ____________ 18___, and ____________ 18___.

Chron.	Meridian Distance.	Observed Mean Daily Temperature, in Interval between the Observations.		Remarks.
		Date.	Mean Temp.	
	h m s			
Z.				
A.				
B.				
C.				
&c.				
&c.				

Arithmetic mean h m s or ° ′ ″

Estimated mean h m s or ° ′ ″

rejecting [any particular chronometers to be mentioned, for reasons specified].

Chronometers (1) days (2) range (3)

Errors and rates at determined by

at by

General direction of ship's head during the voyage

Approved, A. B., Captain. C. D., Officer in charge of Chronometers.

Notes.

(1.) Number of chronometers employed in estimated mean to be specified.
(2.) Number of days between the epochs of the observation.
(3.) The range is the greatest difference of the resulting meridian distance by any two chronometers.

N.B. Every meridian distance measured is to be reported separately in this form. The column of Remarks is to contain anything noteworthy, likely to have affected the chronometers or the value of the meridian distance.

Form for the popular Exhibition of the Results of Meridian Distances.

Abstract of the Meridian Distances measured in H.M.S. ______________ during the year 18___
[or, from ______________ 18___ to ______________ 18___].

A. B., Esq., Captain.

No.	Date.	Places.	Points to which the Mer. Dist. refers, or has been reduced.	Meridian Distance.	Position.	No. of Chron.	Range.	Temperature.	Direction of Ship's Head.	Interval Rate Obs.	Interval Time Obs.	Remarks.
I.	1850. Oct.	Sincapore Hong Kong	Battery Point Victoria Cathedral	$0^h\ 41^m\ 19^s{\cdot}02$ $10°\ 19'\ 45''{\cdot}3$	E.	5	$15^s{\cdot}69$	80° to 75	N.N.E.	$27^d{\cdot}00$	$27^d{\cdot}00$	Single Alt. A.M. Obs. at Sincapore; Eq. Alt. at Hong Kong.
II.	Nov.	Hong Kong Amoy	Victoria Cathedral Citadel	$0^h\ 15^m\ 50^s{\cdot}76$ $3°\ 57'\ 42''{\cdot}4$	E.	5	7·88	60 to 50	N.N.E.	27·00	13·00	
III.	Nov. Dec.	Amoy Shanghai	Citadel Consular Flagstaff	$0^h\ 13^m\ 25^s{\cdot}20$ $3°\ 21'\ 18''{\cdot}0$	E.	5	9·57	50 to 43	N.N.E.	27·00	14·00	Single Alt. A.M. Obs. at Amoy. Single Alt. A.M. Obs. at Hong Kong; Eq. Alt. at Shanghai.
IV.	Nov. Dec.	Hong Kong Shanghai	Victoria Cathedral Consular Flagstaff	$0^h\ 29^m\ 16^s{\cdot}08$ $7°\ 19'\ 1''{\cdot}2$	E.	5	16·49	60 to 43	N.N.E.	27·00	27·00	
V.	Dec. Jan. 1851.	Shanghai Hong Kong	Consular Flagstaff Victoria Cathedral	$0^h\ 29^m\ 16^s{\cdot}47$ $7°\ 19'\ 7''{\cdot}0$	W.	5	12·50	43 to 57	S.S.W.	22·5	22·5	Eq. Alt. Obs. at both places.

Observations.—The meridian distances, as recorded, are supposed to be corrected by the computer for any reduction required from the actual spot of observation to the station to which they are reduced, so as to be in an immediate condition for comparison with the results of other observers. The letters E and W, in the column headed "Position," indicate the position of the second-named place with reference to the first; that is, in point of fact, the direction in which the meridian distance was measured.

By the "Range" is meant the greatest difference between the results by any two of the chronometers.

The column of "Remarks" may contain a brief notice of the nature of the observations employed; also of any material facts affecting the value of the measurements.

It is recommended that the meridian distances should be recorded in chronological order, and that the several measurements should be numbered consecutively.

Table exhibiting the Rates of the Chronometers of H.M.S. "Fly," employed Surveying on the Coasts of Australia, during a period of Four Years, from March 1842 *to April* 1846.

Place.	Date.	Z.	A.	B.	C.	D.	E.	F.	G.	H.	I.	K.	L.	M.T. Fahr.
		s	s	s	s	s	s	s	s	s	s	s	s	
Greenwich Observatory	1842. 7 III.	+2·77	−5·21	−4·76	...	−0·67	...	...	+1·06	−5·16	−3·00	+1·93	−1·51	
Devonport (Mr. Cox's)	15 ,,	...	...	...	+0·90	...	−0·10	+3·20	...	...	...	...	...	
Devonport (on bd. ship)	21 ,,	1·09	5·66	4·91	0·82	0·70	0·24	2·77	1·47	4·49	4·38	1·09	0·55	49°
Falmouth	10 IV.	0·50	5·33	5·18	0·38	0·35	+0·42	2·25	1·20	4·84	5·09	0·66	0·38	50
Madeira	23 ,,	0·20	4·33	1·38	2·04	+0·47	0·75	2·77	1·88	2·60	5·10	1·49	+0·79	60
Teneriffe.................	3 V.	−0·58	4·29	0·00	2·11	−0·38	0·27	2·60	2·68	1·35	5·53	1·01	2·13	67
Simon's Town (C.G.H.)	26 VI.	0·15	3·05	−2·11	2·33	+0·08	1·37	3·35	3·21	1·75	4·55	−0·79	2·67	62
Ditto	14 VII.	+1·15	1·78	0·56	3·67	1·57	2·68	4·37	4·49	4·87	3·54	0·82	3·08	62
Hobarton (V.D. Land)	5 IX.	0·62	2·16	3·11	2·54	1·01	2·64	3·96	3·87	8·21	4·29	0·26	1·62	61
Ditto	4 X.	0·70	1·51	1·43	3·33	1·20	1·94	4·08	3·78	9·88	4·10	0·31	1·96	60
Port Arthur	10 ,,	1·35	1·22	0·40	4·00	2·05	3·10	4·50	4·35	10·35	3·30	0·70	2·60	58
Sydney (N.S. Wales)...	26 ,,	0·35	1·60	0·48	4·18	1·71	2·63	4·30	4·48	8·55	3·89	0·05	3·35	62
Ditto	22 XI.	0·00	1·85	1·73	4·25	1·70	1·05	3·78	4·55	3·57	4·82	0·60	3·60	71
Port Stephen...........	30 ,,	0·00	1·50	1·29	4·50	1·60	2·00	4·05	4·05	3·20	4·40	0·65	2·65	70
Ditto	16 XII.	−0·15	1·58	1·11	5·08	1·71	1·67	4·18	4·71	3·72	4·30	0·60	3·28	73
Sandy Cape	1843. 1 I.	0·46	2·06	2·66	5·07	1·34	0·81	3·78	4·34	0·13	5·23	1·39	3·27	76
Port Bowen	23 II.	1·92	2·95	6·26	5·60	1·07	1·33	3·80	4·15	3·68	5·98	2·42	3·93	79
West Hill	11 III.	1·67	2·77	5·96	6·28	1·55	1·30	4·10	4·47	2·78	6·12	2·27	3·47	81
Cape Upstart............	7 IV.	1·75	2·34	2·85	On shore	1·72	1·15	4·40	4·70	2·46	6·22	2·29	3·48	80
Ditto	16 V.	0·93	1·15	1·58	On shore	3·07	2·82	5·65	5·30	1·83	4·90	1·48	3·47	75
Goold Island	31 ,,	0·60	0·96	+0·08	6·71	3·29	3·15	5·85	5·27	0·57	5·47	1·46	3·68	74
Sir C. Hardy's Islands	17 VII.	1·80	1·74	0·88	7·78	2·90	2·87	5·64	5·03	1·21	6·05	2·23	3·26	77
Port Essington	25 VIII.	2·25	1·75	−0·35	9·26	2·61	2·56	6·09	5·29	3·02	6·39	2·11	3·81	79
Coepang, Timor.	5 IX.	2·20	0·99	+0·87	9·21	3·01	4·07	6·31	5·64	3·20	6·69	2·36	3·90	82
Fremantle (Swan Riv.)	15 X.	0·10	stop.	9·50	9·18	4·18	5·50	7·89	5·95	+0·21	4·81	0·55	4·78	63
Hobarton	19 XI.	+0·27	...	6·01	10·20	5·13	5·20	8·47	6·17	−2·45	4·37	0·36	6·30	62
Ditto	1844. 7 I.	0·13	+3·96	4·38	10·41	4·88	5·50	8·29	6·43	1·32	4·29	0·34	6·31	66
Sydney	2 II.	−1·14	2·41	stop.	10·26	4·99	4·70	7·92	6·67	1·89	5·07	0·92	4·93	73
Ditto	29 ,,	1·26	1·74	...	10·28	4·85	4·49	7·91	6·71	2·16	5·19	0·16	5·44	72
Ditto	24 III.	0·78	1·49	...	10·51	5·05	4·84	8·37	6·79	1·38	4·69	0·68	6·24	69

Table exhibiting the Rates of the Chronometers of H.M.S. "Fly," employed Surveying on the Coasts of Australia, during a period of Four Years, from March 1842 to April 1846.

Place.	Date.	Z.	A.	B.	C.	D.	E.	F.	G.	H.	I.	K.	L.	M.T. Fahr.
		— s	+ s	+ s	+ s	+ s	+ s	+ s	+ s	— s	— s	— s	+ s	°
Port Stephen	1844. 5 IV.	0·68	1·61	2·38	10·60	4·51	5·07	8·38	6·60	2·33	4·89	1·08	6·53	70
Sandy Cape	21 ,,	1·64	0·43	1·69	11·03	...	5·23	7·96	6·83	0·02	4·64	0·87	6·29	73
Cape Upstart	6 V.	2·54	— 0·79	— 0·46	10·94	3·04	3·96	7·60	6·49	0·30	6·16	0·91	5·74	76
Sir C. Hardy's Islands	25 ,,	3·79	1·39	2·29	10·83	3·76	3·41	7·11	6·08	1·04	6·44	1·97	5·46	80
Raine's Island	7 VI.	3·73	1·47	2·92	10·81	3·97	3·67	7·13	6·20	1·45	6·56	1·67	6·13	81
Ditto	4 VII.	3·90	0·00	4·68	On shore	4·06	3·55	7·30	6·12	...	6·25	1·30	5·81	80
Sir C. Hardy's Islands	17 ,,	3·92	+ 1·55	5·18		4·57	4·35	7·88	6·32	0·55	6·55	1·27	6·15	80
Ditto	30 VIII.	4·76	1·90	5·90		4·74	3·99	8·14	6·86	0·91	6·44	1·54	6·36	79
Port Essington	2 X.	4·85	3·40	5·10	11·70	4·90	4·23	8·25	7·03	+ 0·30	6·87	2·08	7·30	84
Sourabaya (I. of Java)	31 ,,	5·95	3·20	4·30	11·36	4·59	4·16	7·80	6·76	0·13	7·39	2·29	6·75	87
Ditto	22 XI.	6·15	On shore	4·45	11·65	4·71	4·45	7·95	6·84	On sh.	7·39	2·71	6·72	86
Ditto	13 XII.	6·34		4·62	11·98	4·72	4·35	8·27	7·00		7·18	2·57	7·36	86
Ditto	1845. 10 I.	6·68		5·01	12·52	5·49	4·85	8·97	7·27	— 0·06	6·71	3·18	6·95	83
Port Essington	2 II.	7·82	0·92	6·25	12·68	5·00	3·93	8·33	6·80	0·55	7·45	3·67	7·62	87
Cape York	20 ,,	8·00	0·84	6·64	12·10	4·96	4·36	8·52	6·62	1·32	7·78	4·08	6·66	86
Oomaga I. (Torres Str.)	22 III.	8·27	1·31	7·50	12·79	4·87	4·03	8·83	6·87	0·81	7·81	4·51	5·89	85
Darnley Island	3 IV.	7·73	0·90	6·89	12·95	5·61	4·87	9·36	8·06	0·16	7·54	4·38	6·73	84
Bramble Key	7 V.	6·80	0·11	5·54	13·59	5·94	5·67	10·27	7·86	3·70	6·84	3·15	7·55	83
Darnley Island	25 ,,	7·34	0·50	5·87	13·01	5·42	5·33	9·76	7·14	4·65	7·38	3·01	7·60	81
Port Essington	17 VI.	6·69	0·59	4·93	14·26	6·04	5·39	10·51	8·54	...	7·06	3·31	8·91	81
Malacca	16 VII.	7·90	0·04	5·53	13·84	6·13	5·10	10·33	7·47	...	7·16	4·22	8·30	84
Sincapore	21 ,,	7·81	0·03	5·52	13·79	5·83	5·38	10·12	7·47	...	7·24	4·21	8·27	84
Ditto	1 VIII.	7·46	...	5·61	13·58	6·22	5·37	10·38	7·80	4·46	6·73	4·38	8·24	83
Sydney	1 X.	4·04	3·16	1·64	13·46	8·02	6·20	13·20	7·68	4·80	4·80	1·28	10·86	66
Ditto	5 XII.	5·81	2·17	3·51	15·16	Supplied to another ship	Supplied to another ship	12·96	Supplied to another ship	On sh.	Supplied to another ship	2·84	9·71	72
Ditto	17 ,,	6·05	2·05	3·78	14·20			12·76				2·55	8·77	75
Port Phillip	1846. 5 I.	8·68	0·50	8·73	14·43			12·99		4·38		1·18	9·63	69
Glenelg (S. Australia)	22 ,,	8·51	1·04	6·42	14·27			12·85		3·60		1·38	10·16	69
Fremantle (Swan River)	16 II.	8·93	1·85	5·35	15·06			13·02		1·53		1·75	10·00	74
Simon's Town (C.G.H.)	10 IV.	6·85	2·41	3·11	14·39			13·55		2·97		0·91	10·51	65

Remarks on the preceding Table.

Numbers and Makers of the foregoing Chronometers:—

Z	No. 652	Murray	[2 days]
A	3327	French	[2 days]
B	4300	French	[1 day]
C	201	Johnson	[2 days]
D	$\frac{814}{2148}$	Cotterell	[2 days]
E	$\frac{915}{18279}$	Litherland	[2 days]
F	1571	Dent	[2 days]
G	2382	Parkinson & Frodsham	[2 days]
H	6279	Porthouse	[Pocket]
I	640	Murray	[1 day]
K	918	Murray	[1 day]
L	2313	Parkinson & Frodsham	[Pocket]

All these chronometers were the property of Her Majesty's Government, except chron. C, which belonged to Captain Blackwood.

Z was employed as the "standard;" H was always employed as an "assistant," and L frequently so; C was also frequently taken on shore for the purposes of astronomical observation.

The other chronometers (excepting A and B) were never moved from the "chronometer room," from the time they were received on board at Devonport, in March 1842, till the ship's return there in June 1846, except when the "Fly" was hove down for repairs at Sydney in October 1845.

Chronometers A and B having stopped on various occasions were repaired at Hobarton and Sydney. The continuity of their performances was thus interrupted.

The rates were always determined by observations, made on shore, with the artificial horizon; and usually by the method of equal altitudes.

A critical examination of the preceding table, which exhibits the actual performances of the chronometers during a period of four years, affords some instructive results.

Chronometers C, D, E, F, G, and L, exhibit with a surprising degree of regularity the tendency which, as we have elsewhere remarked (*ante*, p. 41), many chronometers have to a gradual acceleration of rate, as time advances, since they were last adjusted.

The changes from their *initial* rates at the commencement of the voyage to their *final* rates at its close were,—

Chron. C	from	$+0^s.90$	to	$+14^s.39$
D		-0.67		$+8.02$
E		-0.10		$+6.20$
F		$+3.20$		$+13.55$
G		$+1.06$		$+7.68$
L		-1.51		$+10.51$

The variation of their rates, notwithstanding minute fluctuations, being always constantly progressive; and in a gaining direction.

Chronometers A and B, until the periods when they stopped, exhibited the same tendency; respectively altering from $-5^s.21$ to $-0^s.99$, and from $-4^s.76$ to $+4^s.38$.

After they were repaired, their rates, never very steady, yielded anomalous results, which it is difficult to reconcile with any regular law.

In chronometers Z, H, I, and K, this tendency is not observable.

The table also affords a marked illustration on two occasions of the tendency to an increase of gaining rate, caused by a decrease of temperature.

First, in September and October 1843, when the ship, proceeding from Timor to the Swan River, altered the temperature from 82° to 63°; a change accompanied by a marked acceleration of rate in all the chronometers except A, C, and G.

Secondly, in Aug. and Oct. 1845, when the ship passed from Sincapore to Sydney, experiencing a fluctuation of temperature from 83° to 66°; in which, as before, all the chronometers, except A, C, G, and H, exhibited a decided acceleration of rate.

It is also worthy of note, that although at different periods the rates of the several chronometers underwent considerable changes, yet that these changes were for the most part gradual and progressive, and amid all their fluctuations in value at different periods, yet that they, on the whole, exhibit a sufficient approximation towards stability of condition, even at the termination of the voyage, to justify, during short periods of time, the general assumption of the theory of uniform

and equable variation of rate in proportion to the time, which we have adopted as the basis of our investigations in the preceding pages.

A table similar to the above, exhibiting in chronological order an abstract of the rates of the chronometers as determined from time to time by observation, should always be kept at the commencement of the "Chronometer Journal;" since, by affording an historic record of the performances of the several chronometers, it is eminently calculated to aid the judgment of the observer, and to assist him in rightly estimating the values of his results.

THE END.

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